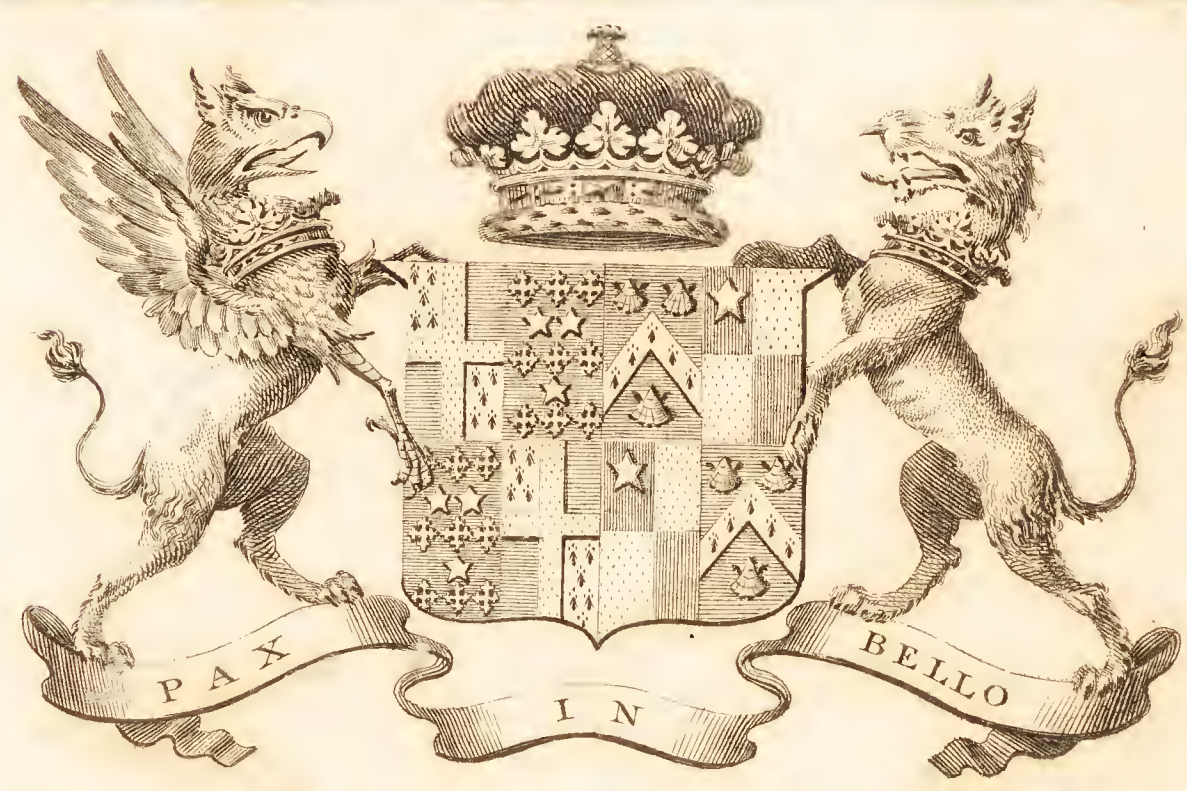




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AN
ACCOUNT
OF THE
TRIGONOMETRICAL OPERATION, WHEREBY THE
DISTANCE BETWEEN THE MERIDIANS OF THE
OBSERVATORIES OF GREENWICH AND PARIS
HAS BEEN DETERMINED.

By Major-general WILLIAM ROY, F. R. S. and A. S.

From the PHILOSOPHICAL TRANSACTIONS.

A



A N A C C O U N T, &c.

Read before the ROYAL SOCIETY, Feb. 25, 1790.

I N T R O D U C T I O N.

THE trigonometrical operation which becomes the subject of the present Paper, had its commencement, as will be remembered, in the measurement of a base on Hounslow-Heath in 1784, an account of which was given to the Royal Society in the following year.

On the completion of that first part of the business, it was little expected, that nearly three full years would have elapsed before, even in this country, an instrument could be obtained for taking the angles!

In the spring of 1787, there were indeed appearances, that Mr. RAMSDEN would have enabled us to embrace the early part of the season, by proceeding with the execution of the main design; and therefore Sir JOSEPH BANKS had opened (through the official intercourse of his Majesty's Secretary of State, the Marquis of CARMARTHEN, with the Ambassador at the Court of France) a correspondence with the Academy of Sciences, regarding the co-operation expected on their part, for connecting the triangles which we were now preparing to
extend

extend along the English coast, with those formerly executed on the coast of France, opposite to Dover. And Dr. BLAGDEN (who for this purpose consented to lay aside his intention of making a tour through Germany that summer) had engaged to assist in the business, on the appointment of the Royal Society, whenever we should be enabled to assign any probable time, for the different parties to repair to their respective coasts, for the aforesaid co-operation.

About the same time likewise, a Paper was laid before the Royal Society, intended as a sketch of the mode proposed to be followed in carrying the scheme into execution; for which purpose it was accompanied with a general map, shewing nearly the disposition of the triangles, and containing also various investigations concerning the figure of the earth, whereon, it is hoped, the result of the present operation will throw some additional light.

For several months of the spring and summer of 1787, Mr. RAMSDEN had been seriously at work in endeavouring to finish the instrument. Not having employed a sufficient number of workmen upon it at the outset, it was now evident, that he had even deceived himself, by leaving too much to be done at the latter end. At length, however, the instrument was produced, and placed on the 31st of July at the station near Hampton Poor-house, on the very spot where, about thirty-five months before, the measurement of the base had been compleated.

By commencing an operation of this nature, at so advanced a season of the year, it was sufficiently obvious, that only very faint hopes could be entertained of bringing it to a conclusion before the bad weather would set in. But it being of much importance to get the triangles, which extend across the
Channel,

Channel, at all events executed, it was therefore proposed to Comte DE CASSINI, who by this time had been appointed by the Academy of Sciences to superintend their part of the business, that he should fix the time that might suit him best for our meeting on the coast; that we would then discontinue the operation to the westward, and, having in concert executed the coast triangles, we would resume the inland parts of our own series at some more convenient opportunity.

This proposition being readily acceded to by Comte DE CASSINI, the 20th of September was appointed for our repairing to the coasts of Dover and Calais respectively.

In the mean time our operation was continued, with all imaginable care and assiduity, through the first ten stations of the series of triangles from Hampton Poor-house to that at Wrotham Hill inclusively.

The instrument, and the various parts of the apparatus, were then removed to Dover, at which place Mess. DE CASSINI, MECHAIN, and LE GENDRE, three distinguished Members of the Academy of Sciences, arrived on the 23d of September.

In the course of two days that these Gentlemen honoured us with their company at Dover (and we regretted exceedingly that the lateness of the season did not admit of our enjoying that pleasure for a much longer period) every thing was settled in the most amicable manner possible, with regard to the times of reciprocal observation.

A great number of white lights, fitted for long distances, and several reverberatory lamps had been previously provided. Having been supplied with such a proportion of the lights as seemed necessary for their side of the channel, and one of the lamps, the French Gentlemen departed for Calais on the 25th, accompanied by Dr. BLAGDEN, who attended them during the

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time

time of the co-operation, until it was finally closed on the 17th of October.

For the greater part of the time, the weather was extremely bad; nevertheless, on the particular nights when the most important observations on our side were made, namely, those at Dover and Fairlight Down, the nights happened very fortunately to be favourable, so as to enable us to intersect, with great accuracy, the two distant points on the French coast of *Blancnez* and *Montlambert* *, and thereby to establish for ever, the triangular connection between the two countries.

The Duke of RICHMOND, Master General of his Majesty's Ordnance, had, in the most liberal manner possible, given every assistance to the operation (from that great department over which he presides with so much honour to himself and advantage to the publick) by furnishing an officer and a detachment of artillery-men for the work; ordering the laboratory at Woolwich † to supply whatever fire-works might be wanted for signals; and temporary scaffolds to be erected at Greenwich Observatory, Shooter's Hill, and Dover Castle, for the reception of the instrument. But what was still of more importance than any of these, his Grace had permitted Lieut. FIDDES (one of the engineers on the survey then under my direction) to be employed, in the summers of 1786 and 1787, in making a very accurate plan of that part of Romney Marsh where the base of verification was to be measured. In a country so much intersected by ditches, and where there were so many ponds of water to be avoided, without such a plan raised before-hand,

* The name of this hill is vulgarly pronounced *Boulemborg*, and it is even written in the same manner in the book, *La Méridienne vérifiée*.

† Major Congreve, of the Royal Artillery, had the management of the lights at Shooter's Hill; and his assistance was found to be most essentially useful.

an operation of so delicate and difficult a nature could not have been effected.

The apparatus for the measurement of the base with the steel chain, notwithstanding the urgency of the case, was not sent to its destination until the end of the first week of October. To Lieut. FIDDES the engineer, was then joined Lieut. BRYCE of the Royal Artillery; and it was not before the beginning of December, that these two gentlemen, with the most unremitting labour and perseverance, were able to accomplish the measurement, as will be seen in the detailed account of that operation given in the first section of this Paper.

In finishing the co-operation with the French Commissioners, at Lydd on the 17th of October, our instrument had now passed through sixteen stations out of twenty-three. There of course remained yet seven stations where it was to be placed, and observations to be made. Eagerly wishing to bring the business to a conclusion, we struggled on through five of the seven. But the weather at length became so tempestuous, that it was utterly impossible to continue it, with any hopes of being able to make satisfactory observations. Perched on the tops of high steeples, such as Lydd and Tenterden, or on heights, such as Hollingborn Hill, we sufficiently experienced, that operations of this sort, where the most important observations could only be made at night, by means of the white lights, should never be undertaken in the latter season.

On the second of November, the instrument was accordingly removed from the top of Hollingborn Hill, and sent to town, leaving the stations on Goudhurst and Frant Churches, both likewise situated on eminences, unoccupied until the ensuing season.

The winter months were employed in calculating the observations that had been made; and from these we were very well enabled to judge to what a degree of accuracy we had arrived in determining the sides and angles: for Frant and Goudhurst, having been intersected from Botley Hill, Wrotham Hill, and Hollingborn Hill; Goudhurst having been observed from Tenterden, and Frant having (contrary to our expectation) been seen and observed from Fairlight Down, we had thereby the certain means of determining very nearly what difference there would be between the measured and computed length of either base as given by the other, although observations had not been made at the two intermediate stations of Goudhurst and Frant. This difference, it was seen, would scarcely amount to one foot, or about $\frac{1}{28000}$ th part of the whole distance. In as far, therefore, as the results of the plane triangles were concerned, we might have proceeded with the computations, and drawn the consequent conclusions, without hesitation, or any risk of sensible error.

But, besides that it might still have been said that the instrument had not been placed at these two stations, there were reasons of a different kind, which rendered it in some degree necessary to place the instrument not only at Goudhurst and Frant, but also at Botley Hill and Folkestone Turnpike, where it had formerly stood.

In 1787, when at the station of St. Ann's Hill, in a very high wind, the box containing the axis level was blown from the scaffold, and unluckily broken. Mr. RAMSDEN replaced it with one not so good as the first; and it was with this second level that the observations of the pole star had been made at Dover Castle. This castle, although lofty, and situated on a high chalk cliff, that raises its northern turret about 466 feet

above low water at spring tides, is nevertheless surrounded on the land side with eminences, at the distance of six or seven miles, still higher than itself. From this circumstance we found it impossible to connect it with the great triangles to the westward, otherwise than by a short side. It was therefore sufficiently obvious, that it would be eligible to make observations of the pole star for determining the difference of longitude, and the convergence of the meridians, at some other intermediate stations between Greenwich and Dover, from whence our longest sides could be distinctly seen. For this purpose none seemed so proper as Botley Hill, Goudhurst, and Folkestone Turnpike. The first of these three is only $171\frac{1}{2}$ feet eastward from the meridian of Greenwich, Goudhurst is about 23 miles south-eastward from the former, and Folkestone Turnpike, the station nearest to Dover, is so situated, that from it can be seen the end of the base of verification at High Nook, Fairlight Down, and other distant stations.

With this object in view, whereon so much depended, we had again the mortification to be thrown into the latter season of 1788.

Besides a better level for the axis of the telescope, the microscope B wanted to be better supported. Another sort of clamp, also an eye-piece, with a diagonal prism for observations near the zenith, or for those of the pole star in high latitudes, were necessary improvements, which might have been executed in a short space of time. With these alterations the instrument was at last returned, but so late, that it could not be placed on Goudhurst Steeple till the 9th of August, 1788.

The observations at Goudhurst, Frant, Botley Hill, and Folkestone Turnpike, having been finished early in September,
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the instrument was brought back to town, in the neighbourhood of which it was employed for three days for the following purpose.

In 1787, when at the stations of Hundred Acres, Norwood, Greenwich, and Shooter's Hill, we had only been able to determine, in a satisfactory manner, two points within the limits of the Capital, namely, St. Paul's Church and Argyll Street, the last by means of the white lights. Bearings of some others, it is true, were obtained; but, in order that these might be intersected in the best manner, it became necessary to place the instrument at one or more stations to the northward of the town.

With the view, therefore, of laying the foundation hereafter for a much more accurate plan of London than could possibly be obtained in any other way, the instrument was placed, first, at Hornsey Hill, to the eastward of Highgate; and, secondly, on Primrose Hill, between London and Hampstead.

Although the weather was rather unfavourable at the time of making the observations from these two new stations; and that the smoke constantly hanging over the town in the latter season impeded us greatly; nevertheless, the former bearings were intersected, and the situations of a considerable number of remarkable steeples within London and its environs, were accurately determined, as will more fully appear in treating of the secondary triangles.

Having thus briefly shewn the order with regard to time in which the recent operation, through its various steps, was progressively carried on and compleated, it is proper that I should mention, that Mr. DALBY, who had been recommended as an assistant, has acquitted himself throughout the whole perfectly to my satisfaction, as a diligent and accurate observer,

observer, as well as an able and indefatigable calculator. This testimony, which is justly due to his merit, joined to the specimen which he gives of his mathematical abilities in the fifth section of this Paper, can scarcely fail of making him better known hereafter; and it is hoped, that he will have opportunities of exerting his talents, by assisting in the continuation of the future operations that are projected and recommended to be carried on in the conclusion of this Memoir; the various parts of which are arranged as follows:

Section First.

Description of the apparatus made use of in the measurement of the base of verification in Romney Marsh, with the hundred-feet steel chain, in the autumn of 1787, with the result of that operation. Reference to be had to Plate I. and II., and also to the table containing the general detail of the measurement.

Section Second.

General description of the great instrument with which the angles in the recent trigonometrical operation were observed; shewing also its various adjustments for practice. Reference to be had to Plate III. a general view of the entire machine; Plate IV. a plan and two sections of it; Plate V. various parts represented on large scales; and Plate VI. the microscopes and eye-pieces.

Section Third.

Description of various articles of machinery made use of in the course of the trigonometrical operation, referred to in Plate VII. Also the distinction of the stations into two sets; those of the second set being referred to in Plate VIII.

Section

Section Fourth.

Calculation of the series of triangles extending from Windsor to Dunkirk, whereby the geodetical distance between the meridians of the Royal Observatories of Greenwich and Paris is determined. Reference to be had to Plate IX.

Section Fifth.

On the difference between horizontal angles on a sphere and spheroid. Plate X.

Section Sixth.

Manner of determining the latitudes of the stations. Application of the pole star observations to computations on different spheres, and also on M. BOUGUER's spheroid, for the determination of the difference of longitude. Ultimate result of the trigonometrical operation, whereby the difference of the meridians of the Royal Observatories of Greenwich and Paris is determined. Plate X.

Section Seventh.

An account of the observations made during the course of the trigonometrical operation for the determination of terrestrial refraction. Plate X.

Section Eighth.

Secondary triangles, subdivided into two sets, for the improvement of the maps of the country, and the plan of the City of London and its environs. Plate XI.

Conclusion,

Containing Propositions for extending trigonometrical operations over Great Britain.

S E C-

S E C T I O N F I R S T.

Description of the apparatus made use of in the measurement of the base of verification in Romney Marsh, with the hundred-foot steel chain, in the autumn of 1787, with the result of that operation. Reference to be had to Plates I. and II.; and also to the table containing the general detail of the measurement.

ARTICLE I. *Preamble.*

IN the account of the measurement of the base on Hounslow Heath in 1784, which appeared in the Philosophical Transactions of the subsequent year, we had occasion to shew, how very accurately distances might be determined by the steel chain, when applied in the ordinary way on the natural surface of the ground, if that surface happened to be tolerably smooth, which was the case in the instance alluded to. By the comparison of the measurement of a length of one thousand feet with the glass rods, and with the chain when used with an apparatus adapted to the purpose, it further appeared, that the difference between the results was so very small as scarcely to be discernible, since it would not have exceeded half an inch on the whole length of that base of 27404.7 feet.

Having always considered the experiment on Hounslow Heath, just now mentioned, as positive proof of the excellency of the chain, it had been resolved on to apply it to the mensuration of the base of verification in Romney Marsh, even if no other reasons had existed to make that choice eligible.

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But besides the danger of having the glass rods broken, in transporting to so great a distance from London, and, on such an event happening, the impossibility of getting them replaced with others at the advanced season of the year in which we were unfortunately thrown with the operation, it was obvious, that in a plain of the breadth of six miles, so much intersected with ditches full of water as Romney Marsh in reality is, the laying of bridges for the tripod stands, which must have been used with the glass rods, would alone have been a very troublesome and tedious operation.

ART. II. *Beech Posts.*

In the first place, about thirty posts made of beech wood, three inches in diameter, and of different lengths, from two feet three inches to three feet six, and a few of them still longer, were provided. They were shod with iron, and each of them carried on its top a cast-iron ferrule, with two dovetails projecting from it; care being taken in driving them into the ground, that the dovetails should stand in or nearly in the direction of the base, as represented by the plan and section of a single post, in the middle part of Plate I. The arrangement of twenty-four of these posts may be seen at the top of the said plate, for the measurement of a portion of the base equal to one hundred yards, or the length of three chains. Sixteen of the posts reckoning from that which stands in the center of the first group, to that which stands in the center of the second, and so on from right to left, were placed at the distance of twenty feet from each other. The first is supposed to coincide with the mouth of the pipe sunk into the earth, at the eastern extremity of the base, at a place called High Nook near Dymchurch; and every fifth post from that towards the
left

left marks the end of a chain. The other eight posts in the arrangement, that is to say, the right and left posts of each of the four groups, are supposed to stand twelve or fifteen inches from those in the center. By referring to the elevation near the top, and the plans and section in the middle part of Plate I. it will be perceived, that these posts, together with certain other iron parts of the apparatus fixed to them, hereafter to be described, support the ends of the coffering for each chain, free and independently of the central posts, to which last the brass scales alone are attached.

ART. III. *Deal Coffers.*

Fifteen deal coffers, numbered from one to fifteen, were necessary for the length of three chains, being five to each. Six of them, that is to say, the first and fifth, the sixth and tenth, the eleventh and fifteenth, being the first and last of each chain, were only nineteen feet four or five inches in length. The other nine, being the three in the middle of each chain, were of the complete length of twenty feet. These coffers perfectly resembled in shape, and nearly in dimensions, the cases of the glass rods, being ten inches broad in the middle, and uniformly of that depth throughout their whole length. But from the middle they became gradually narrower, in a curvilinear manner, towards each end, where they were only two inches wide. The two cheeks or sides were about half an inch thick, and the bottom, which entered into a shallow groove in the middle of the cheeks, was an inch in thickness. Thus the cheeks being thin, bent and applied easily to the bottom, to which they were firmly nailed, and the whole was fortified by small blocks of wood fastened at intervals in the inside, sometimes above and sometimes below the bottom.

From the elevation it will be perceived, that nine or ten inches of the under extremities of the cheeks were cut off, so as to permit the bottom itself to rest on the irons. This construction of the coffers was found to answer very well, that is to say, they were, considering their length, not so heavy as to be unmanageable, at the same time that by their general figure, and particularly the depth of the cheeks, they were entirely prevented from warping.

In addition to the fifteen coffers, just now described, a sixteenth, not represented in the plate, was afterwards prepared at Hythe by Lieut. FIDDES, to be used occasionally, when the end of one chain, and commencement of another, co-incided with a deep ditch or one of the sewers full of water, and where of course it would have been extremely difficult, if not impossible, to have fixed steadily the group of three posts in the usual manner. In this coffer there was a double or false bottom, with grooves adapted for the purpose; and the brass scale, pulley, &c. were removed from the irons, and placed on this bottom.

ART. IV. *Apparatus of cast iron, &c. for the ends of the Chain.*

By referring to the plate, where the several parts of the apparatus for the extremities of the chain are represented in plan and section, by a scale equal to one-fourth of their real dimensions, it will appear, that the cast-iron pieces were of two different forms, one long, and the other short; but both applied in the same manner, on the ferrules binding the tops of the posts, as has been already mentioned. Of the long kind there were in all fifteen or sixteen, that is to say, one for each post in a length of three chains. Each iron had two clamps on its
under

under side, which being slackened, it was placed on its ferrule at right angles to the line of measurement; and being turned round 90° , the dovetails of the ferrule, standing originally in the direction of the base, came within the clamps, which were then tightened by four screws, turned with square keys adapted to the purpose.

It is sufficiently obvious, that so many irons, with such a number of screws to each, could not fail of rendering this operation tedious! The business would have been greatly expedited if there had been only two such screws, one on each end, in a middle situation; and, instead of the four screws, there should have been four steady pins, entering easily into holes prepared for them in the under side. A short groove, of two or three inches in length, in each extremity of the bottom, would, on this supposition, have been necessary to suffer the square heads of the screws to pass; and it will be readily conceived, that the thickness of the bottom would have effectually secured the chain from touching them, prevented the mutilation of its handles, and saved much loss of time. Indeed the same purpose might have been effected, but not so advantageously, by laying the original four screws lower in the iron, which its thickness easily admitted of. Finally, in order to avoid such like inconveniencies in future, there is still one imperfection more, which it is incumbent on me to remark, namely, that cast-iron ferrules will not answer; for the force that was found to be necessary to drive the posts into the ground, burst almost the whole of them, so that before the operation was completed, they were obliged to be replaced with others made of hammered iron, forged for the purpose.

Of the short irons only three were necessary, one for each end of the chain, and a spare one in case of accident. They
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were placed, turned, and clamped on the ferrules, in all respects similarly with those of the long kind. By inspection of the plate it will be seen, that each of them carried on its surface a brass scale of six inches in length, divided into inches and quarters, and moveable in a slide, either backwards or forwards, by a finger-screw adapted to the right-hand end.

The right-hand post of each group is called the *drawing-post*, because the iron fixed on its top carries a small apparatus of brass, which being connected with the flat iron rod and hooks formerly used at Hounslow Heath, for a like purpose, lays hold of the rear handle of the chain, and draws it back until zero co-incides with the point of commencement. The left-hand post in each group is called the *weight-post*, because it carried a brass pulley, over which a weight of 28 lbs. was hung by a small rope attached to the hooks that laid hold of the front handle of the chain. This weight acting against the force of the screw at the other end, the chain was thereby kept perfectly straight in the coffers, and constantly in the same degree of tension, until some certain division (the nearest for instance) of the scale could be brought, by means of the screw, accurately to coincide with 100 feet at the front end. That division, whatever it might be, was of course registered in the field book of the operation, together with the true temperature of the chain, as shewn by five thermometers, one being laid for that purpose in each coffer, and secured with white cloth from the sun's rays, as occasion might require.

Fifteen coffers were always arranged on the ground at the same time, comprehending a space of the base equal to the length of three chains or 100 yards. The extremities of the first chain having been accurately transferred, in the manner above mentioned, to the brass scales on the tops of the central posts,

posts, and these remaining firm and motionless, as being wholly unconnected with any other parts of the apparatus, the chain was then moved forward into the second set of coffer, where the thermometers were also placed. In the mean time, the first set of coffer now vacated, with their posts, &c. were carried on and arranged in the front, for the measurement of the second 100 feet; and so on continually with the others in succession.

ART. V. *Of the survey of Romney Marsh previously to the measurement of the Base.*

In the introduction to this Paper it has been mentioned, that the Duke of RICHMOND had permitted Lieut. FIDDES, of the Royal Engineers, to be employed in 1786 and 1787 in raising a plan of that part of the Marsh where, on examination, it should be found, that the base of verification might be the best executed. In justice to that officer, I consider it as incumbent on me to say, that it was impossible for any person to fulfil the duties entrusted to him better than he did, either in the course of the survey, or subsequent measurement of the base, whereof he also had the direction. The general instructions given to him were, that after having by a base of his own determined certain triangles in the neighbourhood of Dymchurch, Ruckinge, and Romney, by way of foundation for his work, he should preserve Ruckinge as the point whereon the *allignement* of the great base was to be directed, and vary the position of that end next the sea-wall in such a manner as to meet with the fewest local obstructions to the measurement between the two extremities. By inspection of the plan Plate II. which comprehends a tract of country of two miles in breadth, one on each side of the base line, it will be perceived, that

that besides the numberless ditches with which this singular plain is intersected, and which it was impossible to avoid crossing, there is almost in every field a watering pond for the cattle, many of them of considerable depth. Nevertheless, so very attentive had Mr. FIDDES been to the accuracy of his survey, that he was enabled, after several trials of other directions, to run a line from High Nook on Dymchurch Seawall, upon the small spire of Ruckinge Church, of the length of nearly six miles, without interfering with any one of the watering ponds, or meeting with any other local obstruction of consequence. So very minute was he in his remarks, and so accurate in the situation of particular trees, that in tracing his line with the telescope, he managed so as to avoid them all, a few insignificant bushes excepted; which I believe to be an instance of exactness scarcely to be equalled.

ART. VI. *Pipes sunk in the ground.*

Permission having previously been obtained from the proprietors of the soil, pipes were sunk into the ground at the two extremities of the base, and also one on Allington Knoll, which last point with Lydd Church * form that side of one of the great triangles depending on the base on Hounslow

* It will be perceived, that several of the names of places differ in their orthography, from that whereby they were expressed in the plan of the intended triangles given in the Paper of 1787. This has been done, on procuring better information in that respect than had formerly been obtained. Mr. COBB, of Lydd, an ingenious gentleman, well acquainted with Romney Marsh, was so obliging as to present me with a manuscript map of that singular plain, compiled by himself from actual surveys, where the names and boundaries of the *waterings*, and many other curious particulars, are very distinctly expressed.—Our plan of the base has therefore derived advantage by adhering to such respectable authority.

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Heath, to be first verified by the measurement of this new base. Every field is surrounded with a ditch, in cleaning of which the earth and mud are continually thrown out on each side, whereby flat dykes are gradually formed on either side. That the measurement might be carried on as nearly as possible in the same plane, that is to say, about fifteen or eighteen inches above the common surface, therefore, narrow grooves were cut in these flat dykes, which the different farmers readily consented to without murmuring. Here it is to be observed, that there was no occasion for levelling the line, Romney Marsh having been formerly covered by the sea, and a considerable part of it, particularly towards the bottom of the range of hills that separate it from the Wealds of Kent, being still lower than the sea at high water, would again be overflowed by it, if much care and expence were not annually bestowed in securing and repairing the dykes, whereby it is protected. Thus the line of the base may be considered as an inclined plane, descending gradually about five feet from the mouth of High Nook pipe to within 246 yards of the Ruckinge end, where the ground in that direction seems to be the lowest. Thence it rises comparatively suddenly, about fifteen feet, to the mouth of the pipe situated in a small field immediately adjoining to Ruckinge Church-yard.

ART. VII. Result of the measurement.

Lieut. FIDDES, in the course of his trigonometrical survey, and of the different measurements he had actually made of the line with a common iron chain, which from time to time was compared with standard rods of deal, had determined the total length of the base within a few feet of the truth, before the ultimate operation began. He had likewise driven into the ground, at the end of every

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thousand

thousand feet, a strong picket, which were numbered 1, 2, 3, &c. from the pipe at High Nook to the 28th near Ruckinge. In all this preparatory part of the business he had no other assistants than the artillery-men of his surveying party. But for the ultimate determination, it being absolutely necessary that he should have the aid of some person in whom he could confide for the management of the operation in general, and particularly for the adjustment of the scale at one end of the chain, while he himself was adjusting that at the other; therefore Lieut. BRYCE, of the Royal Artillery (now of the Corps of Royal Engineers), an attentive officer and excellent mathematician, was left with him for those essential purposes. These two gentlemen began the operation on the 15th of October, and, after experiencing many difficulties arising from the badness of the weather in that late season of the year, and the defectiveness of the apparatus, it was only by dint of great labour, and the utmost perseverance, that they were enabled to accomplish the measurement on the 4th of December following.

The annexed general table of the base, which contains five columns, shews the progress that was made in the work from day to day. The first column contains the date; the second, the spaces measured each day, reckoned by hundreds of yards, and denoted in the general plan by strong dots; the third shews the temperature of the chain deduced from the mean of fifteen thermometers, five for each chain; the fourth expresses the difference of temperature above or below 62° of FAHRENHEIT; and the fifth shews the correction answering to that difference, additive to the apparent length with the sign +, and subtractive from it with the sign -.

From

Feet. In. Pts.

From inspection of the table it will appear, that the total apparent length of the base, as given directly by the steel chain, was $9512\frac{2454}{10000}$ yards; which are equal to

28536 . 8.835

But when the new points, at the distance of twenty-five feet from each other, were laid off on the chain in Mr. RAMSDEN's shop from the original points on the great plank of New-England deal, the temperature was 55° , that is, 7° below 62° ; wherefore the contraction of the chain by 1° of FAHRENHEIT being $=0.00763$ in. this $\times 7^{\circ} \times 285.37$ chains $=15.242$ in. is the reduction for the total contraction below 62° , to be taken from the apparent length; which are equal to

1 . 3.242

The apparent length is likewise to be lessened by the excess of the corrections with the sign — above those with the sign + in the annexed table; because the temperature of the chain, when actually applied to the measurement, being so much below 62° , the apparent length became thereby too great by 30.65 inches, which are equal to

2 . 6.65

To be deducted also from the apparent length, the reduction on two hypotenusal distances, measured at the Ruckinge extremity of the base, which is suddenly elevated above the lowest part fifteen feet, amounting to

0 . 3.023

The sum of these three reductions, to be taken from the apparent length, amounts to

4 . 0.915

D 2

And

Feet. In. Pts.

And consequently there remains for the
length - - - - - 28532 . 7.92

But when the chain was adjusted in Mr. RAMSDEN's shop, as above-mentioned, the temperature was 55° . Being then carried into St. James's Church-yard, its length was laid off on brass pins inserted into the stone coping of the church-yard wall, for the purpose of comparison on any future occasion, at which time the temperature had changed to $55^{\circ}\frac{1}{2}$. After the measurement in Romney Marsh had been finished, the chain in the temperature of 39° , being stretched out on the wall, its length was found to fall short of the original points on the brass pins $\frac{1.03}{1000}$ of an inch. Now, $55^{\circ}.5 - 39^{\circ} = 16^{\circ}.5$, and $16^{\circ}.5 \times 0.00763 = 0.126$ in.; hence $0.126 - \frac{1.03}{1000} = 0.023$ in. is the space which the chain had only lengthened during an operation which continued above six weeks; and one-half of this space, viz. 0.0115 multiplied by 285.37 chains is $= 3.282$ in., the correction to be added to the apparent length for the wear of the chain during the operation - - - - -

+ 0 . 3.282

Whence the length becomes - - - - -

28532 . 11.202

Lastly, the base is to be shortened for its height of $15\frac{1}{2}$ feet above the mean level of the sea, supposed to be 6 feet 8 inches above low-water spring tides at High Nook, which gives for the reduction - - - - -

- 0 . 0.166

And

And hence there remains for the ultimate or true length of the base of verification, in the temperature of 62° of FAHRENHEIT's thermometer, being the heat to which that on Hounslow Heath was reduced, - -

28532.11.036.

Which make 28532.92 feet.

ART. VIII. *Remarks on the comparative accuracy of the two bases.*

With regard to the accuracy of the measurement of this base, compared with that executed on Hounslow Heath in 1784, from the infinite pains and care bestowed in both operations, it is very difficult to say, to which the preference should be given. The expansion of glass being so much less than that of steel, if manageable glass rods of equal length with the chain could have been obtained; then, as far as that single circumstance might have affected the result, a measurement made with such glass rods would undoubtedly have deserved the preference to one with the steel chain. But when it is considered, that the expansion of steel was determined by the pyrometer with the same care as that of glass; that the wear of the chain is so very small, as we have shewn it to be, in six weeks use; that coffers were laid for it, and its length transferred by means of the brass scales to the tops of immovable posts; that, in the present case, there was but one-fifth part of the error arising from faulty co-incidences as with the twenty-feet glass rods; on this view of the matter, the preference seems to be due to the measurement by the steel chain, supposing always the error in excess, caused by the deviation from the *allignement* horizontally or vertically, to have affected both equally.

As

As a proof that the expansion of the chain was accurately determined, I shall close this section with a remark repeatedly made by the two gentlemen entrusted with the execution of this last measurement. At the close of each day's work, the two scales marking the extremities of the last chain (after registering the divisions of co-incidence) were left upon their respective posts until the next morning. They were secured during the night, from being disturbed by cattle, with a certain number of the spare posts driven into the ground around them. A tent was also pitched between the two, where some men of the party constantly lay, by way of a guard for the whole apparatus. On the recommencement of the operation the subsequent morning, the chain being applied anew to the brass scales; if the temperature continued the same, the co-incidences were found to be equally accurate as on the preceding night; but if it had changed one or two degrees, the chain never failed unequivocally to shew it, by falling short of the divisions on the scales, if the cold had increased, or by over-reaching them if it had diminished.

Finally, with respect to the subject of these bases, it is here to be remarked, that the base of verification in Romney Marsh makes with the meridian of the pipe at High Nook an angle of $54^{\circ} 28' 56''\frac{1}{2}$ north-westward; and *that* on Hounslow Heath makes with the meridian of the pipe at Hampton Poor-house an angle of $44^{\circ} 41' 49''$, also north-westward.

[To face page 26.]

GENERAL TABLE of the Measurement of the BASE of VERIFICATION in ROMNEY MARSH, executed in the Autumn of 1787, whereby the apparent Length is found to be $9512\frac{2454}{10000}$ Yards, and the true, or corrected Length in the Temperature of 62° , $28532\frac{92}{1000}$ Feet.

Days.	Spaces measured.	Temperature.		Correction for the difference.	Days.	Spaces measured.	Temperature.		Correction for the difference.	Days.	Spaces measured.	Temperature.		Correction for the difference.	Days.	Spaces measured.	Temperature.		Correction for the difference.	Days.	Spaces measured.	Temperature.		Correction for the difference.
		Mean by 15 Therm.	diff. from 62°.				Mean by 15 Therm.	diff. from 62°.				Mean by 15 Therm.	diff. from 62°.				Mean by 15 Therm.	diff. from 62°.				Mean by 15 Therm.	diff. from 62°.	
	Yards.			In. Parts.		Yards.			In. Parts.		Yards.			In. Parts.		Yards.			In. Parts.		Yards.			In. Parts.
Oct.					Oct.					Nov.														
15	100	54.7	- 7.3	0.16710		2100	65.0	+ 3.0	+ 0.06867		4100	55.2	- 6.8	0.15565		6100	42.1	- 19.9	0.45551		8100	36.3	- 25.7	0.58827
16	200	62.7	+ 0.7	+ 0.01602		2200	64.1	+ 2.1	+ 0.04807	10	4200	55.3	- 6.7	0.15336	Nov.	6200	39.3	- 22.7	0.51960		8200	39.2	- 22.8	0.52189
	300	61.3	- 0.7	0.01602		2300	56.7	- 5.3	0.12132		4300	53.6	- 8.4	0.19228	21	6300	43.3	- 18.7	0.42804	Nov.	8300	39.5	- 22.5	0.51502
17	400	57.0	- 5.0	0.11445	30	2400	58.7	- 3.3	0.07554		4400	49.0	- 13.0	0.29757		6400	46.5	- 15.5	0.35479	29	8400	34.8	- 27.2	0.62261
	500	52.2	- 9.8	0.22432		2500	59.5	- 2.5	0.05722	12	4500	50.1	- 11.9	0.27239		6500	45.6	- 16.4	0.37540		8500	40.5	- 21.5	0.49213
	600	53.6	- 8.4	0.19228	31	2600	57.3	- 4.7	0.10758		4600	47.9	- 14.1	0.32275	22	6600	42.5	- 19.5	0.44635		8600	42.3	- 19.7	0.45093
20	700	46.8	- 15.2	0.34793		2700	54.6	- 7.4	0.16939	13	4700	44.7	- 17.3	0.39600		6700	42.2	- 19.8	0.45322	30	8700	30.5	- 31.5	0.72103
	800	58.9	- 3.1	0.07096	Nov.	2800	53.9	- 8.1	0.18541		4800	44.8	- 17.2	0.39371		6800	41.2	- 20.8	0.47611		8800	44.3	- 17.7	0.40515
23	900	53.9	- 8.1	0.18541	1	2900	49.0	- 13.0	0.29757		4900	41.3	- 20.7	0.47382	23	6900	39.8	- 22.2	0.50816	Dec.	8900	46.0	- 16.0	0.36624
	1000	55.3	- 6.7	0.15336		3000	54.0	- 8.0	0.18312	14	5000	41.8	- 20.2	0.46238		7000	39.0	- 23.0	0.52647	1	9000	43.0	- 19.0	0.43491
24	1100	55.7	- 6.3	0.14421		3100	50.9	- 11.1	0.25408		5100	42.9	- 19.1	0.43720		7100	37.7	- 24.3	0.55623		9100	45.6	- 16.4	0.37540
	1200	50.0	- 12.0	0.27468	2	3200	49.1	- 12.9	0.29528	15	5200	45.3	- 16.7	0.38226	24	7200	36.2	- 25.8	0.59056		9200	48.7	- 13.3	0.30444
	1300	55.2	- 6.8	0.15565		3300	50.4	- 11.6	0.26552		5300	44.1	- 17.9	0.40973		7300	42.1	- 19.9	0.45551	3	9300	41.1	- 20.9	0.47840
26	1400	59.1	- 2.9	0.06638		3400	48.5	- 13.5	0.30901		5400	40.4	- 21.6	0.49442		7400	40.5	- 21.5	0.49213		9400	46.9	- 15.1	0.34564
	1500	60.0	- 2.0	0.04578		3500	42.6	- 19.4	0.44407	16	5500	41.5	- 20.5	0.46924	26	7500	35.2	- 26.8	0.61345	4	9512.2454	48.4	- 13.6	0.34942
	1600	59.1	- 2.9	0.06638	5	3600	52.3	- 9.7	0.22203		5600	44.8	- 17.2	0.39371		7600	39.8	- 22.2	0.50816					
27	1700	63.1	+ 1.1	+ 0.02518		3700	53.0	- 9.0	0.20601		5700	44.6	- 17.4	0.39829		7700	38.5	- 23.5	0.53791					- 6.97148
	1800	68.1	+ 6.1	+ 0.13963		3800	52.4	- 9.6	0.21974	17	5800	40.6	- 21.4	0.48985	27	7800	33.6	- 28.4	0.65098					- 10.14802
	1900	57.9	- 4.1	0.09385		3900	47.3	- 14.7	0.33648		5900	39.4	- 22.6	0.51731		7900	38.9	- 23.1	0.52876					- 7.58574
29	2000	60.8	- 1.2	0.02747	7	4000	55.6	- 6.4	0.14650		6000	41.3	- 20.7	0.47382	28	8000	32.7	- 29.3	0.67068					- 3.77913
				- 2.16540					- 3.77913					- 7.58574					- 10.14802					- 2.16540
																				Total Correction		- 30.64977		

From the above Table it appears, that the total apparent length of the Base, as given immediately by the Steel Chain, was $9512\frac{2454}{10000}$ yards, which are equal to $28536\frac{8.835}{1000}$ Feet. In.
The corrections in the above Table, and others specified in the Text, being subtracted from the apparent length, — — — — — 3 9.799
There remains, for the true length of the base in the temperature of 62° of Fahrenheit, — — — — — $28532\frac{11.036}{1000}$ Feet. In.
equal to — $28532\frac{92}{1000}$ Feet.

SECTION

SECTION SECOND.

General Description of the great instrument with which the angles, in the recent trigonometrical operation, were observed; shewing also its various adjustments for practice. Reference to be had to Plate III. a general view of the entire machine; Plate IV. a plan and two sections; Plate V. various parts represented to large scales; and Plate VI. the microscopes and eye-pieces.

ARTICLE I. Preamble.

IN endeavouring to describe the curious instrument made use of for observing the angles in the recent trigonometrical operation, it has been judged best to confine ourselves to the principal parts, without entering into any detail of the *minutiæ*: for even to have mentioned these, with the almost infinite number of little screws that serve to unite them into one entire machine, which could only have been done by references to a multitude of great and small Roman and Greek characters, would have been a disgusting labour. By the help of the four plates which this description refers to, and which have been executed with great care, that fewer words might suffice, it is hoped, that the instrument may be understood by two classes of people for whom it is chiefly intended; first, by those who being possessed of such a machine would wish to make themselves masters of its use; and, secondly, by such ingenious artists as would attempt to construct such another; for these last, in particular, the parts that are of brass, of bell-metal,

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or of steel, are distinguished from each other. And here it is necessary to observe, that the plates must not only be frequently consulted, but also attentively considered, and repeatedly compared with each other, in the course of this description.

ART. II. *General view of the instrument.*

It is a brass circle, three feet in diameter, and may be called a great theodolite, rendered extremely perfect; having this advantage in particular, which common theodolites have not, that its transit telescope can be nicely adjusted by inversion on its supports; that is to say, it can be turned upside down, in the same manner that transit-instruments are, in fixed observatories.

The circle is attached by ten conical tubes, as so many *radii*, to a large vertical, conical hollow axis of twenty-four inches in height, which may be called the exterior axis. Within the base of this hollow axis, a collar of cast steel is strongly driven; and on its top there is inserted a thick bell-metal plate, with sloping cheeks, which, by means of five screws, can be raised or depressed a little.

The instrument rests on three feet, which are firmly united to each other at the place where they branch off, by a strong circular plate of bell-metal, upon which rises another vertical hollow cone, of less size than the former, being included within it, and is therefore called the interior axis. On its top is inserted a cast-steel pivot, with sloping cheeks, passing through the bell-metal plate on the top of the exterior axis, the cheeks of the one being nicely ground to fit the cheeks of the other. The bell-metal base of this interior axis is in like manner ground to fit the cast-steel collar in the base of that which is without it. Thus the circle being lifted up by two
men

men laying hold of its *radii*, and the exterior being placed upon the interior axis, the cheeks at the top being at the same time adjusted to their proper bearing, it turns round very smoothly, and is perfectly, or at least as to sense, free from any central shake. This mode of centering is one of the chief excellencies of the instrument. From the use that has been made of it both years, it seems not to have suffered in the least; and it is perhaps the only construction that could have answered for a machine of such magnitude, undergoing so many quick transitions from place to place, and so often raised to high situations without any risk of being thereby hurt.

ART. III. *Mahogany Planes under the instrument.*

By inspection of the plates, but more particularly the III^d, and the section towards the right hand in the Vth, it will be seen, that there are three planes of mahogany under the metal parts of the instrument; namely, that which forms the top of the stand, which, although a square of about three feet four inches at bottom, becomes, by the separation of the legs, an octagon at top. In the center there is a circular opening of nine inches diameter, the use of which will appear hereafter. Over the top of the stand lies another plane of mahogany, likewise an octagon, of somewhat greater dimensions than the former, with a circular curb running around it, about half an inch within the planes of its sides. This octagon hath in its center an open conical socket of brass, three inches in diameter; and on four of its opposite sides there are fixed four strong brass screws, one on each side, which acting against pieces of brass inlaid into the opposite sides of the top of the stand, the octagon plane, with every thing that rests on it, may thereby be moved in four opposite directions, until the plummet sus-

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pending

pended from the center of the instrument above, is accurately brought to co-incide with the point marking the station underneath. The third or uppermost plane of mahogany is in fact a part of the instrument itself, being at all times by screws or otherwise united to it, and carrying the handles whereby it is lifted out for use, or in again into its case, to be transported from place to place. In the middle of this plane or bottom to the instrument, there is another conical brass socket, of three inches and a quarter in diameter, fitted to slip over and turn easily on that in the center of the octagon underneath. In the brass cover of this socket, there is a very small hole concentric with the instrument, to suffer the thread or wire to pass, which suspends the plummet; and in the view, Plate III. may be seen another small box that contains the thread, with a winch-handle for raising or lowering the plummet, according as the height of the instrument above the station on the ground, or edifice where it stands, may require.

ART. IV. *Feet Screws for levelling the instrument.*

By attending to the group representing the front elevation of the feet screws, with its side nuts, in the right hand upward angle of Plate V. it will appear, that they are slackened, which is always the case before the instrument is levelled, to give room for that operation by the action of the screws. This being done, the side nuts are brought to press gently on the horizontal plate that embraces the whole group, and thereby keeps the instrument as it were united to the mahogany until some fresh adjustment becomes necessary. When the instrument is to be put into its case, then the feet are let down, and by the side nuts the horizontal plate is brought to press strongly on

on the whole group, whereby it is kept perfectly fast and secure from motion in carrying from one situation to another.

ART. V. *Blocks of box wood and conical rollers under the feet screws.*

By referring also to Plate V. it will appear, that directly below each foot there is fixed to the lower surface of the mahogany a small block of box wood, curvilinear in the direction of its motion. On these three blocks rests the whole weight of the instrument, which nevertheless can be moved circularly on them alone. But to render the motion perfectly easy, three conical brass rollers, placed somewhat nearer to the center, are, by means of their respective springs and regulating screws, brought to act and receive such a proportion of the weight as it may be necessary to lay upon them. The head of one of these screws, which give more or less action to the rollers, may be seen at D in the principal view of the instrument Plate III. as well as in the plan and section Plate V.

ART. VI. *Screws giving motion to the whole instrument.*

By examining attentively the general view of the instrument may be seen, in two positions, the great screw with the flat ivory head, whereby the entire machine received a circular motion. In one, it is attached to the curbs, as when in use in 1787; in the other, it is laid upon the mahogany bottom, as was the case the same year every time it was carried to a new situation. But this ivory-headed screw having been found to act by jerks in moving so great a weight, and consequently to be troublesome in adjusting the instrument to the fixed point, or that of commencement in measuring angles; it was therefore laid aside in 1788, and another apparatus or clamp was

adapted for the same purpose. This last may be seen attached to the curbs, as represented towards the right hand of Plate V. It consists of a brass cock, fixed to, and projecting outwards from, the curb of the instrument; which cock is acted upon by two screws working on the opposite sides against it, and which are clamped to the curb of the octagon.

ART. VII. *Mahogany Balustrade and Cover.*

The curb, whereon the three feet of the instrument rest, carries a balustrade of mahogany fitted to receive, on the top thereof, a mahogany cover, nowhere represented except in the two sections in Plate IV. In this cover there are only four small openings (besides that which allows the great vertical axis to pass), viz. one for each vertical microscope, one for the clamp of the circle, and one for the socket of the Hook's-joint. The two last are less than the former. At the same time that this cover effectually secures the circle with its cones from dirt and from accidents, it serves conveniently for laying the Hook's-joint upon, or any thing that may be constantly wanted near at hand; but more particularly for placing the lanterns used at night for reading off the divisions on the limb of the instrument that come immediately under the vertical microscopes.

ART. VIII. *Achromatic Telescopes.*

Two achromatic telescopes, each of thirty-six inches focal length, with double object-glasses of two inches and a half aperture, belong to the instrument. They are excellent of their kind, and are furnished with eye-pieces of different magnifying powers, for erect as well as inverted vision. The lower telescope lies exactly under the center of the instrument, and is directed through one of the openings of the balustrade. Being
only

only used for terrestrial objects, it requires but a small elevation or depression, and therefore is only supplied with a short axis of seventeen inches in length, supported by braces attached to the feet. The eye end of this telescope is purposely made heavier than the object end; and resting on an horizontal arm, that is raised or depressed by rack-work, it is thereby readily brought to bear, and remain very steadily, upon its object. The rack-work may be seen in the view of the instrument, and also on the left side of the right hand section in Plate IV. But there is a small horizontal motion that can be given to the right hand end of the axis of this telescope, which is effected by means of a handle inserted through the vacancy of the balustrades, and placed on a dovetail at E, which could not be shewn in the plate. Thus the instrument being nicely levelled, the upper telescope at zero, and likewise on its object, the lower telescope, by the help of this adjustment, is brought accurately to the same object, supposed to be the point of commencement, or that from which angles are measured.

By referring to Plates III. and IV. and likewise to the section on the left side of Plate V. it will be seen, that a horizontal bar extends across the top of the vertical axis, supported by two side braces that spring from the cone, about one-third of its height above the plane of the instrument. The horizontal bar carries the Y's or supports, in which the pivots of the upper telescope move. They are of such height as to permit a semicircle of six inches radius, attached to the axis of the transit, to pass freely, and consequently the telescope to be directed to the sun or stars in high elevations, but not to be brought to the zenith. The arc of excess of the semicircle likewise admits of several degrees of depression being measured thereon.

ART.

ART. IX. *Spirit Levels.*

The instrument has two very good spirit levels, that are fitted with the several means of adjustment, as is usual in such cases, the detail of which it is unnecessary here to enter into. The first or axis level, because it is only applied on the axis of the telescope, is that whereby it is set horizontal, as in the ordinary transit instrument; and it is likewise used for placing the conical axis truly vertical, so that the instrument may turn round without sensible alteration of the level, previously to observations of the pole star, or of other heavenly bodies.

The second, or elevation level, is that whereby the telescope is brought to be truly horizontal, when angles of elevation or depression are to be taken. At such times it is suspended on a rod attached to the outside of the telescope, to whose axis of vision the rod, by adjustment, can be made parallel, as will readily be conceived, by observing the representation of these parts in the right hand section of Plate IV.

When the angles of elevation or depression to be determined are very small, they are measured by the motion of an horizontal wire in the focus of the eye-glass of the telescope; but when great, their quantity is measured by the arc of motion of the semicircle, as shewn by its proper horizontal microscope.

The elevation level is likewise made use of for levelling the instrument when horizontal angles only are to be taken, for which purpose it is suspended on two pins, which are seen projecting from the horizontal bar in the plan, and one of them in each of the sections in Plate IV. This was the ordinary position of the elevation level when the angles of the triangles

triangles were observed, and thereby it was easily seen in the course of the operation, whether the instrument had suffered any change to render a re-adjustment necessary.

ART. X. *Lanterns for the Illumination of the Wires.*

The axis of the transit telescope is hollow, and in the middle there is placed, at an angle of 45° with the axis of vision, a perforated elliptical illuminator for throwing light on the wires in night observations. The light is communicated from a small lantern attached to the horizontal bar at its junction with the brace, directly opposite to the end of the axis, which has a bit of thin glass placed before it to prevent dust from entering. There is another such lantern for the lower telescope, not however represented in the plate. As the light given by these lanterns was found to be rather too weak, especially that for the upper telescope, therefore it was customary in practice to illuminate the wires, by holding up frontwise one of those seen in the section in Plate IV. against the end of the axis of the upper telescope, when directed to the pole star. The same method was used by presenting it obliquely to the object-glass of the lower telescope, when it became necessary to examine whether the intersection of the wires continued without sensible variation on a reverberatory lamp, commonly placed twelve or fifteen miles off, and sometimes even at the great distance of twenty or twenty-four miles.

ART. XI. *Lanterns for throwing light on the Divisions of the instrument.*

Besides the two small lanterns for illuminating the wires of the telescopes in night observations, two larger ones may be seen, as already mentioned, standing on the mahogany cover in the
section

section in Plate IV. used for reading off the divisions of the instrument, under the vertical microscopes. The front of one of these is shewn, and the back, or that to which the handle is fixed, of the other. Their narrow sides are presented towards the microscopes, there being in each a silvered reflector of copper at FF; and opposite to it, at GG, a screen of talc or transparent oiled paper. The light from a wax candle being thrown on the reflectors, and thence back again through the screens, on the divisions of the instrument under the microscopes, these could be very distinctly read off and registered: for the light communicated in this way was very strong, at the same time that the glare of it, which otherwise would have been disagreeable to the sight, was removed by passing through the screen.

ART. XII. *Arms projecting from the bell-metal plate under the plane of the instrument.*

By referring to Plates III. and IV. but more particularly the latter, it will be perceived, that there are three flat arms, strongly fixed by screws to the edge of the circular bell-metal plate, forming, as has been already mentioned, the basis of the interior vertical axis. These arms, which are also firmly braced to the feet of the instrument, rise gradually as they project outwards towards the circumference of the circle, whose radius they exceed about an inch and a quarter, and their extremities are about an inch lower than its upper surface. One arm, lying directly over one of the feet, is that to which are attached the wheels and screw moved by the Hook's-joint, and also the clamp of the circle, as represented in Plate V. The other two arms, whereof one lies also over a foot, and the other directly opposite to it, become thereby a diameter to the circle,

circle, having their extremities terminated in a kind of blunted triangular figure, forming the bases of pedestals whereon stand the vertical microscopes hereafter to be described. The arms, together with the horizontal bar and braces carrying the transit telescope, are every where pierced, in order to lessen the weight without diminishing the strength of the parts.

ART. 13. *Vertical Microscopes.*

Two vertical microscopes, distinguished A and B, are used for reading off the divisions on the opposite sides of the circle immediately under them. They are exactly of the same construction, and the chief parts of that marked A are represented in their real dimensions towards the left hand of Plate VI.; where, beside the general, may be seen particular plans of the slides, and also that of the pedestal, containing within it the gold tongue, with its axis and screws for adjustment. Next to these plans stand the elevation and optical lines, shewing the position of the glasses with the magnified scale at the bottom.

Each microscope contains two slides, one lying immediately over the other, their contiguous surfaces being in the focus of the eye-glasses. The uppermost, or that nearest the eye, is a very thin plate of brass, to the lower surface of which is attached the fixed wire, having no other motion than what is necessary for adjustment, by the left hand screw to its proper dot, as hereafter to be explained.

The steel slide immediately under the former is made of one entire piece, of sufficient thickness to permit the micrometer screw, of about 72 threads in an inch, to be formed of it. To its upper surface is fixed the moveable wire, which changes its place by the motion of the micrometer head, seen in the plan and elevation towards the right hand. The head is divided

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into

into 60 equal parts, each of which represents one second of angular motion of the telescope. By examining the particular plan of this steel slide, it will be seen, that it is attached by a chain to the spring of a watch, coiled up in the usual manner, within a small barrel adjacent to it in the frame. By this provision no time whatever is lost; the smallest motion of the head being instantly shewn, by a proportionable motion of the wire, to one hand or the other, in the field of the microscope.

It is necessary to remark, that the whole microscope between its pillars can be raised or depressed a little more or less, with regard to the plane of the circle, by the help of two steel levers, seen one on each side of the elevation, which for that purpose are applied in the holes represented above and below the projecting plate that unites the tops of the pillars. By means of this motion, distinctness is obtained at the wires; and by the motion of the proper screw of the object lens, which necessarily follows that given to the whole microscope, the scale is so adjusted as that fifteen revolutions of the head shall move the wire over fifteen minutes, or one grand division, on the limb, equal to nine hundred seconds, each degree on the circle being only divided into four parts. This operation being delicate, requires great patience and many repetitions, before the purpose can be exactly, or even nearly, effected: for at the same time that the fixed wire must bisect the dot on the gold tongue, the moveable wire must also bisect the dot at 180° on the limb, as well as the first notch in the magnified scale at the bottom of the plate, where the minutes in the field of the microscope are represented in the proportion of between fifteen and sixteen to one as painted on the eye of the observer. In this adjustment there is yet another circumstance to be
I attended

attended to, which is, that fixty on the micrometer head should stand nearly vertical, so as to be conveniently seen. A few seconds of inclination to one side or the other are of no moment, because the dart or index being brought to that position, whatever it may be, must at all times remain there without alteration, unless some derangement that may have happened to the instrument, in transporting from one place to another, should have rendered a fresh adjustment necessary. But if, when the wires co-incide with their respective dots and the first notch, fixty on the micrometer head should happen to be underneath, or so far over from the vertex on either side as to be seen with difficulty, then the gold tongue must be moved a little by means of the capstan-headed screws, which act against each other on the opposite extremities of its axis. Thus, by repeated trials, the wished-for object will at length be effected, that is to say, fixty, to which the dart is to be set, will stand in a place easily seen. But it is not to be expected, that each microscope will give just nine hundred seconds for the run of fifteen minutes. Without great loss of time this cannot be done; besides that two observers, of different sights, will adjust the microscopes differently. Accordingly, in 1787, after many trials of the runs in measuring fifteen minutes on the different parts of the limb, microscope A was found to give only 896'', while B gave at a medium 901''. But in 1788, microscope A gave 900'', while B gave no more than 894''. These differences were of course registered and allowed for in the estimation of the angles for computation, whereby any difference between them almost wholly disappeared.

The gold tongue, which is extremely thin, applies very closely to the surface of the circle. In the plan it is supposed to be seen through a thin plate of brass covering the whole

pedestal, and also through a small square plate lying over the former, and fastened to it by three screws. In the under side of this last, there is a cavity for the projecting part of the tongue. This contrivance of the tongue with its dot was to guard against any error that might arise from accidental motion given to the instrument between one observation and another, which from this precaution could never happen, without being immediately discovered: for the wires being adjusted to their dots under the microscopes respectively, if the instrument be then turned round 180° , the wires will reciprocally bisect the dots that were originally opposite to them, and thereby shew, that they are accurately in the diameter of the circle; and so on with regard to any other dots whatever. Hence this becomes the most severe mode of trying the justness of the divisions of the instrument.

ART. XIV. *Manner of reading off angles with the microscopes.*

By attending to the magnified scale at the bottom of the plate, it will appear, that the dot on the gold tongue, which is here inverted, is about one minute to the left of zero, and also of the first notch, with which the moveable wire alone co-incides. Now it will easily be conceived, from what has been said in this description, how readily, as well as accurately, any observation of an angle can be read off with such an instrument; for the degrees and quarters, that is to say, the $15'$, $30'$, or $45'$, being seen with the naked eye, and registered, the value of the fractional space between zero and the last past grand division, seen in the field of the microscope, is obtained by turning the micrometer head until the moveable wire bisects the dot at that grand division. The number of notches towards the right hand passed over on the scale, equal to so many revolutions

tions of the head, are the number of minutes, always less than 15', to be added. If there be no odd seconds, the dart will then stand at 60° on the head; but, if any number of seconds are to be added, the dart will shew, by its position with regard to 60°, what that number is. Thus, by adding the parts together, the measure of the total angle is obtained.

The construction, adjustment, and application of these vertical microscopes have been given more fully, because they form a most essential part of the instrument: for the fixed wire constantly remaining on its dot, the fractional space may be repeatedly measured many times over, if necessary, and a mean result may then be taken. But it rarely happens that two observers, reading off with the opposite microscopes, differ more than half a second from each other at the very first reading. If time therefore permits, and the circumstances of the weather should also be favourable for repeating the observation with the telescope, it is sufficiently obvious to what a wonderful degree of accuracy the measure of angles may in this way be obtained.

ART. XV. *Horizontal Microscopes.*

Besides the two vertical microscopes, applied in the manner that has been described to the measurement of the fractional space in horizontal angles, there is yet another to be mentioned, which is placed horizontally on the bar that carries the transit telescope, and is directed to the divisions on the semi-circle attached to its axis, for the measurement of angles of elevation or depression, as has already been taken notice of. This microscope, which is of the same construction with the others, but larger, being upwards of nine inches in length, is represented in its full dimension in Plate VI. It has, like the others, a slide made of steel, of such thickness as to permit the
micrometer

micrometer screw to be formed of it; and it carries a vertical wire placed in the focus of the eye-glasses, in which position it is moved parallel to itself from left to right, by turning the micrometer head. This slide is also attached to a watch spring which acts in a contrary direction to the head, as in other microscopes of this sort.

Each degree of the femicircle being divided into two parts or 30', and one revolution of the micrometer head moving the wire in the field of the microscope 3'; therefore in 10 revolutions it changes its place half a degree or 30', which are shewn by a scale of 10 notches in the upper part of the field of the microscope, and also represented towards the top of the plate. Each notch corresponds to 3 minutes or 180 seconds, and the head being divided into 3 minutes, and each minute into 12 equal parts, therefore each part is of the value of five seconds.

ART. XVI. *Concerning the Semicircle.*

With regard to the femicircle, which has been repeatedly mentioned in the course of this description, it is yet necessary to make some remarks; and particularly to shew how, by its means, the axis of vision of the telescope, when adjusted, is brought and kept truly horizontal, which is effected in the following manner.

On the opposite sides of the horizontal bar that carries the telescope there are fixed four small, but finely polished bell-metal planes, two on each side, on the right and left of the top of the vertical axis, in such a manner as that the surfaces of the two on either side are directed to or in the same plane with the center of the axis of the telescope. These planes will be best
conceived

conceived by observing attentively the top of the vertical axis in the section towards the right hand of Plate IV. On the edge of the femircircle may likewise be seen a moveable clamp, easily made to flip, with the hand only, around its circumference, and it carries with it a very fine steel screw. When the femicircle is towards the left hand of the telescope, which is its ordinary position, the point of the steel screw rests, or may be made to rest, perpendicularly on the surface of the plane that is on the left of the vertical axis. But when the telescope is inverted in its Y's, or turned upside down, as is the case in adjusting the line of collimation, the semicircle being then on the right of the telescope, and the clamp necessarily brought down, the point of the steel screw accordingly rests perpendicularly on the surface of the plane to the right of the vertical axis. Thus it will be readily conceived, that in adjusting the telescope by the level for elevations, which is then constantly suspended on its proper rod, parallel to the axis of vision, the action of the steel screw on the bell-metal plane serves not only for the adjustment of the telescope in a truly horizontal position, for angles of elevation or depression, by the motion of a wire in the focus of its eye-glass, in the manner hereafter to be described, but also to keep it in that position, by the superior weight of the eye end, rendered so on purpose. By the same means the telescope remains steadily on any object that it may be directed to for intersection, whether above or below the plane of the horizon.

One thing more with regard to the femicircle must be mentioned, namely, that it gives angles of elevation $12''$ too great, and those of depression $12''$ too little. It is very easy to conceive, that this arose from the impossibility of dividing it on the axis of the telescope to which it is fixed, and through the
centers

centers of whose pivots an imaginary line passes that should at the same time have passed through the center of the semicircle. Mr. RAMSDEN took the best method that could be devised to render the excentricity as little as possible. Having framed the semicircle, and screwed it in its place on the axis, he made a steel point firmly fixed to the horizontal bar describe the concentric arcs whereon the divisions were afterwards to come, and then marked the point for zero, when the telescope by adjustment had been brought as nearly horizontal as possible. These previous steps being taken, the semicircle was removed, divided on the engine, and replaced in its original situation. Nevertheless, when the instrument was carried into the field, and scrupulously adjusted, the error was found, as has above been said, $12''$, which of course became the constant quantity to be applied with its proper sign, when angles of elevation or depression were taken.

ART. XVII. *Eye-glasses of the telescopes, and mechanism of the wires in their foci.*

It has been already mentioned, that the telescopes of the instrument are furnished with eye-glasses of different magnifying powers for erect and inverted vision, six for each telescope, as follows, *viz.*

Erect vision.		Inverted vision.	
	N ^o Power.		N ^o Power.
For the lower telescope,	N ^o 1. 58.	N ^o 1. 43.	
	2. 88.	2. 59.	
	3. 117.	3. 87.	
For the upper telescope,	N ^o 1. 54.	N ^o 1. 40.	
	2. 81.	2. 55.	
	3. 108.	3. 80.	
With			

With regard to these eye-glasses, it is only necessary here to mention, that those of the least magnifying powers were found both in day and night observations to answer the best.

In the focus of the eye-glass of the lower telescope there are only two wires crossing each other in acute angles, which are vertical, instead of being placed at right angles, horizontally and vertically, as was the ancient method. Since the lower telescope never moves through more than a few degrees of a vertical arc, the wires require little or no adjustment. Nevertheless this was provided for, by allowing room for a small circular motion of the end-piece, which, when adjusted, is then fastened by its proper screws, and never afterwards needs any alteration.

By referring to the middle part of Plate VI. two representations of the eye end of the upper telescope will be seen, with the eye-piece removed. Five wires are shewn in this end, namely, two that intersect each other in acute angles, similarly to those in the lower telescope; and three that lie horizontally or parallel to each other. Four of these, *viz.* the two that form the acute angles, and the two extreme horizontal wires, are fixed in the focus of the eye-glasses to the farther surface of a thin brass slide, supposed to be seen through the outward brass, and therefore shaded more dark than the rest. This slide, as will be conceived, lies nearest the eye, and is moveable from right to left, and, *vice versa*, horizontally, for the adjustment of the line of collimation, by the insertion of a small mill-head key, on a square pin fitted to receive it, and secured by a socket on the right hand side. The fifth or middlemost horizontal wire is attached to the nearest surface of a steel slide, that lies contiguously to, but beyond the former. It is made of one entire thick piece, like those of the micro-

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scopes,

scopes, to permit the micrometer screw to be formed of it; and it is represented in the uppermost figure attached to a watch spring coiled up in the usual manner.

By the motion of the micrometer head, the slide, and with it the wire, moves upwards or downwards in the field of the telescope, a space equal to half the distance of the extreme wires from each other. This motion above or below the central point, which was made to correspond with the acute intersection of the wires placed in the axis of vision of the telescope, is performed in ten revolutions of the head, as denoted by the motion of the dart, ten divisions upwards or downwards, in the narrow groove seen at the top of the figure.

Now, by the means of this piece of mechanism in the eye-end of the telescope, it will appear sufficiently obvious, that small angles of elevation or depression may be determined with great accuracy, when the value of a certain number of revolutions and parts (the circumference of the head being divided into 100) have been once ascertained by repeated observations of the altitude of any well-defined object taken by the semicircle. Thus it was found, by experiment, that $7\frac{77}{100}$ revolutions of the micrometer head were equal to an angle of elevation or depression of $10' 59''$, or $659''$, on the semicircle. Whence it follows, that one revolution raises or depresses the wire above or below the central point $1' 24''.8134$, or a little more than $84' 81$. And hence a motion of one division on the head raises or depresses the wire nearly $\frac{85}{100}$ ths of a second.

In this manner were determined the reciprocal elevations or depressions of the several stations of the series of triangles with regard to each other.

By observing attentively the four screws represented in the outward end of the telescope, a dotted groove will be seen
under

under the head of each. And in the uppermost figure there appears a flat brass ring, soldered to the inside of the tube about half an inch from the outward end, which carries on its surface four studs to receive the lower extremities of the four screws. Thus the grooves allow room for a small circular motion to be given to the end-piece for the vertical adjustment of the fork of the wires, those that are horizontal being by construction at right angles with it. This being done, the screws are made very fast in the studs below, and thereby the whole machinery of the end-piece is rendered perfectly firm and secure.

There remains yet one piece more to be barely mentioned. It is the prism eye-tube, represented by dotted lines towards the right-hand side of Plate VI. as attached to the eye end of the transit telescope, instead of the common eye-piece with two convex glasses. In leaning over our instrument to observe the pole star, highly elevated in these latitudes, the body is necessarily thrown into an inconvenient fatiguing posture, whereby some risk is run of deranging the instrument, and consequently of making the observations less accurately than when the observer can look directly forward, without bending the body so much. For this purpose, Mr. RAMSDEN promised to supply the prism tube in 1787; but it was only, and with great difficulty, obtained in 1788, by which time Mr. DALBY had accustomed himself to observe very well without it, so that it was never used.

By employing this piece, light is no doubt lost; because the image passes through more glasses before it reaches the eye, than when the common eye-piece is used. But for observations of stars nearer the zenith than the pole star is in our latitudes, it would be indispensably necessary. It would likewise be advantageously

used in looking at the meridian sun in summer, for which purpose it is furnished with dark glasses, placed in a slide moved by rack-work, as may be seen from inspection of the plate. They consist of three prisms, laid close to each other, so as to form, when thus assembled, a parallelopiped. Here the green prism stands nearest to the eye, a dark one farthest from it, and between the two, one of white flint glass, for correction of the refraction which would otherwise take place. It will easily be conceived, from the disposition of the prisms, that the darkest medium is here towards the left; and that it becomes gradually lighter towards the right hand, where a void part in the frame is brought into the field when the stars are observed; or when, from the circumstances of the weather, it may be unnecessary to screen the eye from the sun's rays.

ART. XVIII. *General management of the instrument for observation.*

When the instrument is used on the ground, it is covered from the weather, under a circular tent, eight feet in diameter. Four short piles, hooped and shod with iron, are driven into the earth, and their heads levelled, by laying across from one to the other a mahogany straight ruler, having a spirit level attached to one side of it. The feet of the stand being then placed on piles, are firmly fastened to them by means of long square-headed screws, only one of which may be seen in the view of the instrument, belonging to that foot which stands nearest the eye. By working with the four screws fixed in the octagonal mahogany plane, the plummet suspended from the center of the instrument is brought accurately over the point on the ground that marks the station. The screws of the feet, with the side nuts appertaining to them, are then slackened,

to

to give sufficient room for the adjustment of the instrument, which by them is brought to be level.

ART. XIX. *Adjustment of the axis Level.*

The axis of the upper or transit telescope being brought over any one of the feet, and the circle being clamped, hang the axis level on the pivots or *ansæ* of the telescope, and bring the bubble to the two indexes; then reverse the level, that is, turn it end for end, and note the difference. Bisect this difference, one half by the level's proper adjusting screw, and the other half by that foot-screw only which is in a line with the axis. This operation being repeated until the difference wholly vanishes, the level will be truly adjusted, that is to say, the bubble will rest between the same points in both positions.

ART. XX. *Adjustment of the elevation Level.*

This level being suspended on the rod attached to the outside of the transit telescope, screw the erect eye-tube on, to make that end preponderate. Adjust the bubble to the indexes by the steel finger-screw at the tail of the semicircle's clamp. Reverse the level, and note the difference. Then bisect that difference, and correct one half by the finger screw, and the other half by the proper adjusting screw under the level, and so on repeatedly until the difference wholly vanishes. The level may then be hung on the two pins that project from the horizontal bar which carries the telescope, where, being parallel to the axis level, it will shew when that is removed (as is commonly the case when terrestrial objects only are observed) whether the plane of the instrument suffers any alteration. If this should have happened, the level on the horizontal bar is at all times sufficient to correct it.

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ART. XXI. *To set the vertical Axis perpendicular.*

This may be done by either level, but best with the axis level, which being suspended on its pivots, must be brought parallel with two of the feet of the instrument; and by the screws of these two feet, the bubble is to be brought between its indexes. The circle being then turned round 180° , if the bubble changes its place, half the difference is to be corrected by one of the feet screws, and the other half by two capstan-headed screws, that act against each other, under and belonging to one of the Y's, or supports, in which the pivots rest. When the bubble is found to be just in these two positions, turn the circle 90° , which will necessarily bring the axis over the third foot of the instrument. Then correct any error there may be by that foot screw. In this manner the circle will be made to revolve again and again, without any alteration whatever of the bubble, which shews that the vertical axis is then truly perpendicular to the horizon.

ART. XXII. *To make the line of Collimation in the telescope at right angles with the transverse Axis.*

The pivots resting in their Y's, direct the telescope to some distant well-defined object, and let the circle be clamped. Then reverse the axis, that is, turn the telescope upside down. If the intersection of the wires does not co-incide with the object in both positions, half the difference must be corrected by the motion of the circle with the Hook's-joint, and the other half by the motion of the brass slide in the eye end of the telescope, by applying the milled-head key in the small socket seen on the right hand side in Plate VI. and so repeatedly until the difference wholly disappears.

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ART. XXIII. *To set the Rod on which the elevation level hangs parallel to the line of Collimation.*

The vertical axis being supposed to be nearly vertical, hang the level on its rod, and rectify the bubble by the finger screw of the clamp. Set the horizontal wire on the steel slide, to intersect the center of the oblique wires, and place the dart or index at zero on the micrometer head. Then observe some distant distinct object covered by the horizontal wire. Invert the semicircle, that is, turn the azimuth circle 180° , and the telescope upside down, so as to bring the wire upon or nearly upon the same object. Now, if the level be not right, rectify it by the finger screw at the tail of the clamp. If the telescope does not now accurately cover the same object as in the former position, bisect the difference by the finger screw of the clamp, and then rectify the bubble by the capstan-nuts under one end of the rod. Repeat this operation until the level is right, when the telescope sees the same objects in both positions, and thereby the rod will be brought parallel in altitude to the line of collimation or axis of vision.

The adjustments of the microscopes having been already sufficiently explained, in giving the description of the essential parts of the instrument, it is unnecessary here to repeat them.

ART. XXIV. *Of the weight of the instrument, and mode of transporting it from place to place.*

The instrument, whose description and uses we have here attempted to give in a general way, without reference to its minute parts, by a multitude of different characters, weighed in the whole about 200 lbs. It is contained in two deal boxes;

one of a circular form for the body of the instrument; and the other of an oblong square figure, for the transit telescope. Within this last box there is one of mahogany, that holds all the smaller parts of the apparatus. The stand, steps, stools, pullies, ropes, tent, and canopy for the scaffold, &c. &c. weighed at least as much more. The whole attirail was transported from place to place, in a four-wheeled spring carriage, drawn by two, and sometimes by four horses. The carriage part, originally that of a crane-necked phaeton, was presented, with his usual liberality, by Sir JOSEPH BANKS; and upon it was built a kind of caravan, covered with painted oil-cloth, whereby every thing within was kept dry and secure.

SECTION THIRD.

Description of various articles of machinery made use of in the trigonometrical operation referred to in Plate VII. Also the distinction of the stations into two sets, those of the second set being referred to in Plate VIII.

ARTICLE I. Portable Scaffold.

IN the account of the measurement of the base on Hounslow Heath we have shewn, that the surface of that remarkable plain is not elevated more than fifty or sixty feet above the mean level of the sea. From this small elevation, and the circumstance of its being surrounded, almost on every side, with lofty trees, it was from the beginning sufficiently obvious, that, in order to be enabled to make the observations of the collateral stations

stations from the extremities of the base, it would be absolutely necessary to raise the instrument, by some means or other, to a considerable height above the ground. For this purpose the portable scaffold, whose plan and elevation are represented on the left hand side of Plate VII. was constructed. It consisted, as may be seen, of an inward scaffold for supporting the instrument, and an outward one for the observers, wholly free and independent of each other, the platforms of both being framed about thirty-two feet above the lower ends of the scantlings, which rest on the ground. These being made of squared deal, and the several parts being bolted and screwed together with many iron screws secured by nuts, the whole could be readily taken to pieces, carried in a waggon (for which it made a complete load), and replaced again in any new situation. This scaffold answered very well the purpose for which it was intended; for the step-ladders, or stairs leading to the platform, being attached to the outward frame, the inward one that carried the instrument remained undisturbed by the motion of those who went up and down, or walked around the top. The silk thread, that suspended the plummet, was secured from the effects of the wind by a sort of funnel or trunk, composed of three deals (one side being left open), and so contrived as to be easily turned round to any quarter of the heavens, whereby the open side was always presented to leeward. The instrument was covered from the weather by a canvas canopy, about seven feet square, to which side walls could be hooked for screening it from the wind, as occasion might require. By referring to the elevation it will be seen, that the scaffolds, both outward and inward, might be divided horizontally into two parts, so as to permit the uppermost half alone to be used when it became unnecessary to raise the instru-

ment to a greater height than fifteen or sixteen feet above the ground. The whole together was never made use of, except at the two extremities of the Hounslow Heath base. The uppermost half was applied at three of the stations only, namely, St. Ann's Hill, Botley Hill, and Padlesworth near Dover.

ART. II. *Tripod Ladder.*

Next to the scaffold the plate represents, in plan and section, a tripod ladder, about thirty-five feet in height. It carries on its top a globe lamp, of about one foot in diameter, in which was used a simple ARGAND's burner, of a large size, made for that purpose. The lamp being removed, a socket for a white light might occasionally be substituted in its place; or (as was the case when we observed the station at King's Arbour from St. Ann's Hill) a flag-staff might be added at the top, which was secured in a truly vertical position, by braces fixed to the legs of the ladder underneath. It will be readily conceived, that by a contrivance of this sort a white light could be raised to a considerable height above the ground, if the circumstances at any time had rendered such elevation necessary; and that it could, by the help of a heavy plummet, be always placed in a truly vertical position over the point on the ground marking the station. The globe lamp was found to answer very well for short distances of six or eight miles, when the weather was favourable; but it could not be depended upon in observations of distances that were considerably greater.

ART. III. *Common Flag-staff.*

After the tripod ladder, comes in the plate the plan and elevation of a common flag-staff with its braces, carrying
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likewise

likewise two reverberatory lamps. These two were attached to the same iron bar, at the distance of three feet from each other. They had concave copper reflectors, nine inches in diameter, extremely well polished and silvered. They were intended at first for experiments near London, and were very well seen at the distance of fifteen or sixteen miles. To secure us from any uncertainty that might have arisen, by mistaking other lights for our own, one lamp was placed over the other. But when we came afterwards to be better acquainted with the appearance of these lamps, that precaution was found to be entirely unnecessary; wherefore single reverberatories were provided, with *specula* of ten inches diameter, and they were supplied with still larger burners, which could be seen at the distance of twenty or twenty-four miles. But here it is proper to remark, that these lamps must be carefully watched, especially in exposed windy situations; for if the cotton be drawn out a little too far, they are apt to smoke, whereby the front glass becomes obscure, and therefore must be wiped frequently. They are easily turned on the posts that support them; and were, by the help of a telescope laid on one side, parallel to the axis of the rays (for which a contrivance was provided in the tin work) accurately presented towards the station occupied by the instrument at the time from whence they were to be observed. There was constantly one of these lamps, and sometimes two, at two different stations, burning each night, when we were making observations of the pole star, or white lights of short duration, placed at other distant stations.

ART. IV. *Tripod for White Lights.*

Next after the flag-staff (whereon a socket for white lights could likewise be placed, when the flag itself was removed) is represented a small tripod intended for white lights only. The same socket that fitted the top of the flag-staff, or lamp-post, could be applied to the tripod, by the help of three small sockets foldered for that particular purpose to the sides of the principal one. Deal rods, of five or six feet in height, or hazels cut from the nearest hedge, served as the legs of this stand. The sockets themselves were made of copper, because those of iron would have been dissolved by the sulphur; and the upper part, which was only an inch, or an inch and an half, in height, was square or round, according to the figure of the boxes containing the composition, sometimes of one kind, and sometimes of the other. These white-light tripods, being readily placed by the help of a plummet over the point marking the station, were found to be very convenient on the top of an open hill, or on the leads of a church steeple, as the person attending them could easily light the box with the port-fire, without the aid of a ladder.

ART. V. *Portable Crane.*

On the right hand side of the plate is represented, in plan and section, and by a larger scale than the others, a portable crane for weighing up the instrument to the tops of such towers, church steeples, or other buildings, as became stations in the series of triangles. It was constructed in the Tower of London, and answered very well the purpose for which it was intended, although it might still be improved. Before we
were

were supplied with this crane, we made shift, by the help of a long beam, and a moveable trestle by way of fulcrum for it to rest upon, to get the instrument up to the top of its own proper scaffold, and one that was still higher, erected over the transit room of the Royal Observatory at Greenwich.

ART. VI. *Reasons for changing certain Stations.*

In the course of the trigonometrical operation, the center of the instrument has constantly been brought, even almost to mathematical exactness, over the precise point marking the station, whereby reductions to the center on account of excentricity have been avoided; and the stations have been distinguished, as far as possible, by permanent marks in such a manner, that, while these remain, the center of this or any other instrument may be again brought into the same vertical line. By these means our recent observations may be repeated on any future occasion, and connected with others, which it is to be hoped will be made hereafter: for this operation, the first of its kind in Britain, should only be considered as the foundation or commencement of a series of others, which by degrees will be carried to the remotest parts of the island.

By comparison of the annexed plan of the triangles with that communicated to the Royal Society in 1787, as only a sketch of the scheme then proposed to be carried into execution, it will be perceived, that some few stations are omitted entirely, and others substituted in lieu of some that were then intended to be occupied. Of this last number Hanger-hill Tower has been made use of instead of Kew Pagoda. This last had been proposed on a supposition, that without a scaffold of an enormous height, it would have been impossible to see ~~it~~ from King's Arbour. Nevertheless, after
a good

a good deal of trouble, by cutting off the tops of certain trees, lopping the branches of others, and raising a flag-staff on the center of the scaffold, these two stations were rendered reciprocally visible. By these means we not only avoided making use of Kew Pagoda, which, from the nature of the building, would have been a very incommodious station; but we thereby got rid of Clermont Tower altogether; and thus, instead of two small triangles, one was constituted, larger and better, being nearly equilateral.

In the introduction there has been occasion to take notice of the advantage that was gained by being able to see Frant and Fairlight Down reciprocally. From this circumstance the series from Frant eastward to the base of verification becomes in reality a double one, and consequently affords better means of ascertaining the correctness of the work.

The singularity of the situation of Dover Castle has likewise been mentioned. Instead of two stations near Tatterlees Barn and Barefristan, whereby it was hoped, that Dover Castle might have been connected with the series to the westward, it was found necessary to make use of three stations; one at Padlesworth, one at Folkestone Turnpike, and a third at Swingfield. Thus the side which connects that ancient fort with the other triangles is shorter than was intended. But with such an instrument as ours, and where all the angles of the triangles were observed, no uncertainty arises on that account.

ART. VII. *Distinction of the Stations.*

Having assigned the reasons that rendered it eligible or necessary to change some few of the stations proposed in the original scheme, it only now remains to enumerate,

distinguished

distinguished into two sets. First, those which are permanently marked by pipes sunk in the earth; and, secondly, those where the instrument was elevated to the top of some tower, church steeple, or other building. The plans of the platforms of this last set are given in Plate VIII. along with such dimensions as are necessary to shew, with regard to the side walls, the precise spot over which the center of the instrument was placed. As often as was possible, these situations were further defined, by means of concentric circles described on the leads.

The stations of the first set, marked with pipes, are fourteen in number, *viz.*

Hampton Poor-house,	{	the extremities of Hounslow Heath
King's Arbour,	{	base.
St. Ann's Hill, .		about the middle on the east edge.
Hundred Acres, .		near the west end of the garden.
Norwood, . . .		towards the Croydon end of the heights.
Botley Hill, . . .	{	in a field belonging to Limpsfield
	{	Lodge Farm.
Wrotham Hill . .		in a field belonging to Mr. JOHNSTON.
Hollingborn Hill, .		in a field belonging to Mr. DUPPER.
Fairlight Down, .	{	347 feet southward from the Windmill,
	{	which makes with Fairlight Church,
	{	an angle of $105^{\circ} 53' 20''$.
Ruckinge, . . .	{	the extremities of the base of verifi-
High Nook, . . .	{	cation.
Allington Knoll, .	{	an artificial mount belonging to Sir
	{	JOHN HONEYWOOD.
Padlesworth, . . .	{	eastward from the Church, in the
	{	Broom-field belonging to Mr. BROCK-
	{	MAN.

Folkstone

Folkstone Turnpike . . westward from the Public-house.

The stations of the second set, where the instrument was elevated on buildings, are nine in number, *viz.*

Hanger-hill Tower.

Transit-room of Greenwich Royal Observatory.

North-west turret of Severndroog Castle, on Shooter's Hill.

Swingfield Church Steeple.

North turret of the Keep of Dover Castle.

Lydd Steeple.

Tenterden Steeple.

Goudhurst Steeple.

Frant Steeple.

S E C T I O N F O U R T H.

Calculation of the series of triangles extending from Windsor to Dunkirk, whereby the geodetical distance between the meridians of the Royal Observatories of Greenwich and Paris is determined. Reference to be had to Plate IX.

ARTICLE I. *Excess of the angles of spherical above those of plane Triangles.*

IF the earth, or any considerable portion of its surface, was a perfect plane, an instrument, such as has been formerly described, when applied on that surface, to determine by trigonometrical measurement the extent of the plane part, would every where have its axis parallel to itself; and the sum of the
three

three angles of each of the triangles, into whatever number, great or small, it might be divided, would constantly amount to 180° . But the earth being a sphere or spheroid, it follows, that the same instrument, successively adjusted at each of the stations, will have its axis perpendicular, on a sphere, to an equally curved surface; on a spheroid, to one unequally curved, in either case forming the horizon of the station; and the sum of the three angles of such a spherical or spheroidical triangle must, as is known, always exceed 180° , less or more, in proportion to the lengths of the sides. When the triangles are very small, the excess being of course small cannot possibly be discernible by common instruments. Even the finest, supposing them free from error of division, will scarcely render it perceptible, without the utmost care in making the observations. This will be sufficiently exemplified in the following calculations, where a column is inserted containing the spherical excess; and another for the difference or error between that and the excess of the sum of the observed angles above 180° . From these it will appear, that, notwithstanding the goodness of our instrument, and the pains taken in using it, we have frequently failed in bringing out an excess; and indeed the results have even sometimes been in a small degree defective.

It had been at first proposed to multiply the observations as much as possible, and particularly by successively changing the zero of the instrument to new points (Phil. Trans. 1787, p. 219.), to measure the same angles on different parts of the circle, so as to subdivide any errors that might arise from inaccuracy of division, or shake at the center. This principle, perfectly good in theory, and which was adhered to as far as the circumstances would permit, was nevertheless found, on many occasions, to be impossible in practice, without sacri-

ficing much more time than we could afford, consistently with the engagements entered into with the French Gentlemen, for the co-operation on the Coast. At particular times, especially in hot weather, there was such a tremulous motion or boiling in the air, that it was only during a very short space, chiefly in the mornings and evenings, that the objects were sufficiently distinct to be observed with accuracy. So difficult it is to do any thing perfectly good in this way, that a whole day has frequently been spent, after watching with anxious care, in obtaining a single one that was perfectly satisfactory! At such times as these it would have been absurd to have attempted to change the zero, which always rendered it necessary to re-adjust the instrument by its levels.

In very favourable circumstances of the weather a good observation by day is preferable to one by the white lights at night; because, in the first case, the observer has time at his leisure nicely to bisect a fine flag-staff, and repeatedly to read off the angle; whereas, in the short duration of the burning of the light, he is somewhat hurried, from the fear of losing some of the lights at other distant stations, if two of them happened to come together, which now and then they did, from the irregularity of the rates of the watches of the artillery-men attending at the different stations. It was, however, by the assistance of the white lights only, that the most distant stations could be rendered visible; and there cannot be a doubt that, in great trigonometrical operations of this sort, they will be universally adopted hereafter.

Sometimes an observation has been entirely lost, or at least that which had been obtained was not thought a very good one. In such cases a blank has been left in the column of observed angles, and also in that expressing the error. But no

bad

bad consequence has arisen on that account, there being always such other checks from the collateral stations, as to leave nothing doubtful.

On the whole, although, for the reasons already assigned, we have repeated the observations seldomer than was at first proposed; yet it will obviously appear from the results, and particularly from the near agreement between the measured and computed length of the base of verification, that a few very good observations are greatly preferable to a mean that might perhaps have been obtained of many made in a hurry, which at best would have been but indifferent.

The quantity by which the sum of the three observed angles of spherical triangles should have exceeded 180° was found as follows.

Because the excess of the three angles of a spherical triangle above $180^\circ \times \text{earth's radius} = \text{its area}$, therefore

$\frac{\text{Area}}{\text{Earth's rad.}} = \text{excess above } 180^\circ \text{ in seconds}$, if the area and radius are taken in seconds. Now, 60859.1 fathoms being $= 1^\circ$ on a mean sphere, we get the log. of the feet in a second $= 2.0061743$, and twice this, or 4.0123486 is the log. of the square feet in a square second. Therefore log. area in feet $- 4.0123486 = \text{log. area in seconds}$; and the log. of the earth's radius in seconds being 5.3144251, we have area in feet $- 4.0123486 - 5.3144251 = \text{log. area in feet} - 9.3267737 = \text{log. excess in seconds}$; that is to say, *from the logarithm of the area of the triangle taken as a plane one, in feet, subtract the constant logarithm 9.3267737, and the remainder is the logarithm of the excess above 180° in seconds nearly.*

ART. II. Calculation of the Triangles.

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
		^o ' "	"	"	^o ' "	Feet.
I.	Hanger-hill Tower	42 2 32			42 2 34	
	Hampton Poor-house	67 55 39			67 55 39	
	King's Arbour	70 1 48			70 1 47	
		179 59 59	0.29	-1.29		
	The BASE between Hampton Poor-house and King's Arbour					27404.7
	Hanger-hill Tower from	{ Hampton Poor-house				38461.12
		{ King's Arbour				37922.57
II.	St. Ann's Hill	44 18 51.5			44 18 51.5	
	Hampton Poor-house	61 26 33.1			61 26 33.5	
	King's Arbour	74 14 35			74 14 35	
		179 59 59.6	0.21	-0.61		
	St. Ann's Hill from					37754.25
		{ Hampton Poor-house				34455.8
		{ King's Arbour				

Hence, in the quadrilateral formed by *Hampton Poor-house*, *King's Arbour*, *Hanger-hill Tower*, and *St. Ann's Hill*, making use of the two obtuse angles, as contained within their respective known sides, we have for the mean distance of the points of the acute angles at *Hanger-hill Tower* and *St. Ann's Hill*, expressed by a dotted line in the plan of the triangles, 68897.165 feet.

III.	Wardrobe Tower of Windfor Castle	58 9 58.5	
	King's Arbour	62 40 27.5			62 40 27.5	
	St. Ann's Hill	59 9 14			59 9 14	
			0.25			
	Windfor Castle from	{ King's Arbour				34819.4
		{ St. Ann's Hill				36032.37

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
IV.	Hundred Acres .	53° 58' 35.75"	"	"	53° 58' 36.5"	Feet.
	Hanger-hill Tower	68 24 44			68 24 44	
	St. Ann's Hill .	57 36 39.5			57 36 39.5	
		179 59 59.25	1.08	-1.83		
Hundred Acres from { Hanger-hill Tower					71934.2
St. Ann's Hill	79211.22
V.	Severndroog Castle,	53 31 10			53 31 9.75	
	Shooter's Hill	55 53 44.3			55 53 44	
	Hanger-hill Tower	70 35 6.75			70 35 6.25	
	Hundred Acres .	180 0 1.05	1.18	-0.13		
Severndroog Castle from { Hanger-hill Tower					84376.68
Hundred Acres	74077.66
VI.	Norwood .	107 53 37			107 53 35.75	
	Hanger-hill Tower	26 12 22.5			26 12 23	
	Severndroog Castle	45 54 1.5			45 54 1.25	
		180 0 1	0.44	+0.56		
Norwood from { Hanger-hill Tower					63673.31
Severndroog Castle					39155.15
VII.	Norwood .	88 5 58			88 5 58.07	
	Hanger-hill Tower	29 41 20.75			29 41 21	
	Hundred Acres			62 12 40.93	
			0.53			
Norwood from Hundred Acres					35648.21
VIII.	Tranfit Room, Green-	111 56 50			111 56 50	
	wich Observatory	47 48 14			47 48 13	
	Severndroog Castle	20 14 58			20 14 57	
	Norwood .	180 0 2	0.01	+1.9		
Greenwich Observatory from { Severndroog Castle					14610.58
Norwood	31274.48

N ^o of triangles.	Names of the stations.	Observed Angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
IX.	Botley Hill .	74 37 17.5	"	"	74 37 18	Feet.
	Hundred Acres .	66 0 56.2			66 0 56	
	Severndroog Castle	39 21 46.25			39 21 46	
		179 59 59.95	0.78	-0.88		
	Botley Hill from { Hundred Acres . Severndroog Castle				48726.75 70194.76
X.	Wrotham Hill .	54 25 1			54 25 1.25	
	Botley Hill .	67 53 11			67 53 10.25	
	Severndroog Castle	57 41 49			57 41 48.5	
		180 0 1	1.12	-0.12		
	Wrotham Hill from { Botley Hill . Severndroog Castle				72953.12 79962.13
XI.	Frant .	50 19 19			50 19 18	
	Botley Hill .	57 15 11.25			57 15 11	
	Wrotham Hill .	72 25 31.2			72 25 31	
		180 0 1.45	1.3	+0.15		
	Frant from { Botley Hill . Wrotham Hill				90364.16 79723.57
XII.	Hollingborn Hill			47 18 59	
	Wrotham Hill .	84 12 24.5			84 12 23.5	
	Frant .	48 28 37.5			48 28 37.5	
			1.52			
	Hollingborn Hill from { Wrotham Hill . Frant	81196.58 107897.5
XIII.	Fairlight Down .	48 25 53.5			48 25 55	
	Frant .	79 23 3			79 23 2	
	Hollingborn Hill			52 11 3	
			2.85			
	Fairlight Down from { Frant . Hollingborn Hill				113928.2 141747.1

N ^o of triangle.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
						Feet.
XIV.	Goudhurst .	35 26 32.5	"	"	35 26 34.5	
	Botley Hill .	40 4 42			40 4 42	
	Wrotham Hill .	104 28 44			104 28 43.5	
		179 59 58.5	1.35	-2.85		
	Goudhurst from { Botley Hill . Wrotham Hill				121809.3 80997.43
XV.	Goudhurst .	72 23 32.5			72 23 33.87	
	Frant .	75 33 16			75 33 13.63	
	Wrotham Hill .	32 3 12.8			32 3 12.5	
		180 0 1.3	0.81	+0.49		
	Goudhurst from Frant	44389.68
XVI.	Hollingborn Hill	63 46 44			63 46 47	
	Wrotham Hill .	52 9 11.5			52 9 11	
	Goudhurst .	64 4 3.5			64 4 2	
		179 59 59	1.22	-2.22		
	Hollingborn Hill from Goudhurst	71296.03
XVII.	Tenterden .	67 7 55			67 7 56.46	
	Goudhurst .	68 13 21			68 13 19.5	
	Hollingborn Hill			44 38 44.04	
			0.85			
	Tenterden from { Goudhurst . Hollingborn Hill				54374.66 71855.0
XVIII.	Fairlight Down			35 20 58.42	
	Goudhurst .	49 39 34			49 39 35.77	
	Tenterden .	94 59 26			94 59 25.81	
			0.91			
	Fairlight Down from { Goudhurst . Tenterden				93625.92 1134.73

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances
XIX.	Allington Knoll .	48° 24' 38"	"	"	48° 24' 39"	Feet.
	Hollingborn Hill			40 0 58.96	
	Tenterden .	91 34 23			91 34 22.04	
			1.05			
	Allington Knoll from { Hollingborn Hill Tenterden	96036.45 61775.34
XX.	Lydd			63 14 9.82	
	Allington Knoll .	73 0 27.5			73 0 27	
	Tenterden .	43 45 22			43 45 23.18	
			0.67			
	Lydd from { Allington Knoll Tenterden	47849.27 66166.93
XXI.	Fairlight Down .	54 59 18.5			54 59 17.31	
	Lydd			62 27 50.18	
	Tenterden .	62 32 53			62 32 52.51	
			0.99			
	Fairlight Down from Lydd				71689.73
XXII.	Allington Knoll .	32 59 22.5			32 59 23	
	Lydd .	125 42 0.25			125 42 0	
	Fairlight Down			21 18 37	
			0.33			
	Allington Knoll from Fairlight Down				106922.5
XXIII.	Lydd .	43 20 48.25			43 20 48.5	
	Ruckinge .	48 58 49.75			48 58 49.5	
	High Nook near Dymchurch .	87 40 21.75			87 40 22	
		179 59 59.75	0.21	-0.26		
	The BASE of VERIFICATION between High Nook and Ruckinge				28532.92
	Lydd from { Ruckinge High Nook				41533.89 31362.58

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
XXIV.	Allington Knoll .	91° 27' 20"	"	"	91° 27' 19.5"	Feet.
	Ruckinge .	54 19 17			54 19 18.5	
	High Nook .	34 13 21			34 13 22	
		179 59 58	0.09	-2.09		
	Allington Knoll from { High Nook Ruckinge				23184.93 16052.44

Hence, in the quadrilateral formed by *High Nook*, *Ruckinge*, *Lydd*, and *Allington Knoll*, making use of the two obtuse angles, as contained within their now respective known sides, we have for the mean distance of the points of the acute angles, at *Lydd* and *Allington Knoll*, represented by a dotted line in the plan of the triangles, 47849.27 feet. This distance agrees accurately with the length of the same side in the XXth triangle, as given by the base measured on *Hounslow Heath*. Here however it is to be remarked, that, in order to produce this agreement, the angle at *Hollingborn Hill*, between *Allington Knoll* and *Fairlight Down*, has been made 48° 56' 28" instead of 48° 56' 31"½, being a difference of 3"½+, which, according to observation, it should have been. Had not this reduction been made, the distance between *Allington Knoll* and *Fairlight Down*, being one of the sides of the XXIIId triangle, would have been 106924 feet, that is to say, 1½ foot longer. Now, since this side, compared with the base of verification, bears nearly the proportion of four to one, it follows, that the real difference between the measured length of that base, and its computed length deduced from that on *Hounslow Heath*, seventy miles to the westward, or of

either base with respect to its opposite one, amounts only to about $4\frac{1}{2}$ inches.

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
						Feet.
XXV.	Folkstone Turnpike	24° 17' 6.25"	"	"	24° 17' 6.25"	
	Allington Knoll	76° 1' 54"			76° 1' 53.25"	
	High Nook	79° 41' 0.75"			79° 41' 0.5"	
		180° 0' 1"	0.29	+0.71		
	Folkstone Turnpike from { Allington Knoll High Nook				55461.7 54706.0
XXVI.	Folkstone Turnpike			32° 6' 56.89"	
	Allington Knoll	109° 50' 40"			109° 50' 39.35"	
	Lydd	38° 2' 24"			38° 2' 23.76"	
			0.59			
	Folkstone Turnpike from Lydd				84659.88
XXVII.	Padleworth	108° 9' 34.5"			108° 9' 34.5"	
	High Nook			14° 48' 25.5"	
	Folkstone Turnpike	57° 2' 0"			57° 2' 0"	
			0.16			
	Padleworth from { High Nook Folkstone Turnpike				48303.7 14713.82
XXVIII.	Padleworth	105° 29' 40.5"			105° 29' 40"	
	Lydd	9° 38' 29"			9° 38' 29.36"	
	Folkstone Turnpike			64° 51' 50.64"	
			0.27			
	Padleworth from { Lydd Folkstone Turnpike				79533.34 14713.82
XXIX.	Padleworth	12° 16' 3"			12° 16' 2.65"	
	Lydd	154° 5' 54.75"			154° 5' 54.4"	
	Fairlight Down			13° 38' 2.95"	
			0.59			
	Padleworth from Fairlight Down				186113.0

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
XXX.	Swingfield .	48° 38' 15"	"	"	48° 38' 15"	Feet.
	Padlesworth .	70 54 5.5			70 54 5.5	
	Folkstone Turnpike	60 27 39.5			60 27 39.5	
		180 0 0	0.06	-0.06		
	Swingfield from { Padlesworth Folkstone Turnpike				17056.06 18525.15
XXXI.	Dover Castle, North Turret .	34 39 26.5			34 39 26.5	
	Swingfield .	75 36 40			75 36 40	
	Folkstone .	69 43 53.5			69 43 53.5	
		180 0 0	0.13	-0.13		
	Dover Castle from { Swingfield Folkstone Turnpike				30559.32 31554.58

Hence, in the quadrilateral formed by *Folkstone Turnpike*, *Swingfield*, *Padlesworth*, and the *North Turret of the Keep of Dover Castle*, making use of the two obtuse angles, as contained within their respective known sides, we have for the mean distance of the points of the acute angles at Padlesworth and Dover Castle 42561.18 feet; and hence, in the triangle, *Dover*, *Folkstone Turnpike*, *Padlesworth*, we have the acute angle at Dover $15^{\circ} 18' 44''\frac{1}{2}$, and that at Padlesworth $34^{\circ} 29' 42''\frac{1}{2}$, as were repeatedly observed.

XXXII.	Dover Castle			21 37 55.42	
	Padlesworth .	152 15 25.5			152 15 25.15	
	Fairlight Down			6 6 39.43	
			0.69			
	Dover Castle from Fairlight Down				186113.0

N ^o of triangles.	Names of the stations.	Observed angles.	Spherical excess.	Diff. or error.	Angles corrected for calculation.	Distances.
		° ' "	"	"	° ' "	Feet.
XXXIII.	Dover Castle			87 30 29.58	
	Fairlight Down			43 19 58.52	
	Montlambert			49 9 31.9	
			7.4			
	Montlambert from { Dover Castle	168821.07
					245777.5
XXXIV.	Fairlight Down			25 33 55.02	
	Dover Castle			110 55 29.83	
	Blancnez			43 30 35.15	
			4.78			
	Blancnez from { Fairlight Down	252469.9
					116655.93
XXXV.	Dover Castle .	23 25 0.25			23 25 0.25	
	Montlambert			36 53 18.11	
	Blancnez			119 41 41.64	
			1.84			
	Blancnez from Montlambert				77235.0

In this last triangle, the angle at Blancnez, as determined with great care from a mean of many observations, by the French Academicians, was found to be $119^{\circ} 41' 28''.9$, that is to say, $12''.7$ less than what results from our observations across the Channel. This difference, which is the *maximum* of the error between us in the joint operation, being small, and of no real importance one way or other, with regard to the main point in discussion, since it only varies the distance between Blancnez and Montlambert two or three feet, and the longest sides of the triangles, which connect the two Coasts, by eight or nine; it has therefore been judged best not to

to make any alteration whatever on account of that difference (except as will be mentioned underneath), but to proceed with our own scale of distances for fixing the relative situation of Dunkirk; making use, nevertheless, in the first seven following triangles, from the XXXVIth to the XLIIId inclusive, of the angles as ultimately settled by the late French operations, which Comte DE CASSINI has been so obliging as to supply us with for that purpose. The angles of the XLIIId and XLIVth triangles are taken from M. CASSINI DE THURY'S Book (*La Méridienne vérifiée*); and those of the XLVth triangle result from the combined operation.

In conformity with the exception above alluded to, we have, in the XXXVIth triangle, added $3''$ to our angle observed at Dover between Blancnez and the *flèche* of the spire of Notre Dame at Calais; that is to say, instead of $12^{\circ} 46' 39''$ it has been made $12^{\circ} 46' 42''$, on a supposition, that the spire may overhang so much from the perpendicular towards Blancnez: because the space between the position of the white light on the gallery, and the axis of the spire, being carefully measured by Dr. BLAGDEN, corresponds to an angle of $9''$, whereas the observation gave only a difference of $6''$.

N ^o of triangles.	Names of the stations.	Angles corrected for calculation.	Distances.
			Feet.
XXXVI. {	N.D. at Calais	$47^{\circ} 27' 6''$	
	Blancnez signal	$119^{\circ} 46' 12''$	
	Dover Castle $12^{\circ} 46' 39''$	$12^{\circ} 46' 42''$	
	N.D. at Calais from { Dover Castle		137449.9
	{ Blancnez signal		35023.3
	Excess above $180^{\circ} = 0''.84$.		

N° of triangles.	Names of the stations.	Angles corrected for calculation.	Distances.
			Feet.
XXXVII.	Fiennes signal.	94 26 27.5	
	Blancnez signal	51 18 27.3	
	Montlambert signal	34 15 5.2	
	Fiennes signal from { Blancnez signal Montlambert signal	43600.8 60464.4
XXXVIII.	N.D. at Calais	64 21 43.1	
	Fiennes	46 24 25.2	
	Blancnez	69 13 51.7	
	N.D. at Calais from Fiennes	45219.6
XXXIX.	Watten	27 37 14.8	
	N.D. at Calais	66 30 36.2	
	Fiennes	85 52 9	
	Watten from { Fiennes N.D. at Calais	89453.5 97283.0
XL.	Dunkirk	51 39 12	
	Watten	85 57 46.5	
	N.D. at Calais	42 23 1.5	
	Dunkirk from { N.D. at Calais Watten	123734.8 83616.2
XLI.	Mont-Cassel	63 24 50	
	Dunkirk	42 7 12	
	Watten	74 27 58	
	Mont-Cassel from { Watten Dunkirk	62711.1 90087.5
XLII.	Hondscôte	93 31 34.1	
	Dunkirk	51 7 4.5	
	Mont-Cassel	35 21 21.4	
	Hondscôte from { Mont-Cassel Dunkirk	70260.7 52228.4
XLIII.	Dunes, signal at the east end of the base	81 57 30	
	Dunkirk	48 43 23	
	Hondscôte	49 19 7	
	Signal on the Dunes from { Hondscôte Dunkirk	39641.0 40000.5
XLIV.	Dunes signal	5 19 47	
	Fort Revers, west end of the base	90 17 29	
	Dunkirk	84 22 44	
	Dunkirk from { Signal on the Dunes Fort Revers	40000.5 3715.6
	Fort Revers from the Dunes, the measured base	39808.7
	But this base by measurement was	39801.7
	The difference is	7.0

In

In order to complete the triangular connection between Greenwich and Paris, there remains yet one triangle more (the XLVth) to be given, whereby we shall be enabled to connect the point M near Dunkirk with Dover. For this purpose it is necessary to make some remarks on the Dunkirk base; and also to shew, from the French operations, how the point M is situated with respect to Paris, Dunkirk, and Calais.

M. CASSINI DE THURY, in his book already quoted (p. 23. and 54.) has informed us of the manner in which this base on the Strand near Dunkirk was measured; and that its mean length amounted to 6224.36 toises, which are equal to 39801.7 English feet. Thus it appears, that there is a difference as above stated of seven feet in defect, between the measured and computed length of the last side of a combined series of 44 British and French triangles, depending on a base measured on Hounslow Heath, and verified by another measured in Romney Marsh. But a series of 24 French triangles, founded on a base measured near Paris, and corrected by another executed near Amiens, gives for the length of the same base near Dunkirk 39809.94 English feet, and consequently only an excess of 1.5 inches with regard to our result. This very near agreement in the determination of the same length by two different serieses of triangles, whose extremities are situated at so great a distance from each other, sufficiently proves the excellency of trigonometrical measurement, and shews to what a wonderful degree of accuracy operations of this sort may be brought when fine instruments are made use of, and great care bestowed in the application of them. Doubtless small errors may have arisen in the progress of the work, unseen on both sides; but these falling sometimes one way and sometimes the other, they seem so far to have compensated for, or destroyed each other, that

that their effects have almost wholly disappeared. With regard to the deficiency of seven feet found between the actual measurement with the deal rods, on the Strand near Dunkirk, and the trigonometrical result, it is necessary to call to remembrance what would have happened, if the base on Hounslow Heath had been measured with our deal rods, when in their greatest state of expansion from the moisture they had imbibed. In the volume of Philosophical Transactions for 1785, p. 438. it has been shewn, that our base would thereby have been rendered more than seven feet and a half too short. Now, although the French rods were covered with several coats of oil-paint to prevent their imbibing the salt water, which we are told rested on the Strand at particular places six inches deep; yet it is presumable, that it would be impossible to prevent it from entering by the extremities at the junction of the ferrules, and extending along the fibres, underneath the paint. Hence, in all likelihood, the intended remedy would prove worse than the disease: for the paint might prevent the rods from drying so soon as they otherwise would have done, and thereby the measurement would be given still shorter than if no paint had been applied. Whether this supposition may be thought to be well founded or not, is left to the determination of those who are conversant in matters of this sort. But the curious fact, one way or other, might be ascertained by means of such a steel chain as ours, in the space of one or two days at most. For, supposing the extremities of the base between *Fort Revers* and the *Dunes* to be accurately known, and the *allignement* traced out on the Strand only with camp colours, placed at reasonable distances from each other, and a moveable cord, by the simple application of the chain on the common surface, without any extraordinary apparatus whatever, for-
wards

wards and again backwards, the distance might certainly be determined within a foot of the truth. And hence the importance is obvious of having at all times so accurate and easy a mode of measurement.

On due consideration of all these circumstances, it will not be thought surprising, that in fixing the situation of Dunkirk and the point M near it, where the meridian of the Royal Observatory at Paris intersects a line drawn from thence to N.D. at Calais, the *Dunkirk base*, with the corrections depending upon it, are here rejected; and that the scale of distances furnished by the British triangles is adhered to, as not differing sensibly from the mean result given by the other two French measurements.

From M. DE CASSINI's Book, *La Méridienne Vérifiée*, p. 51. 53. and 56. it appears, that Dunkirk (rejecting the corrections formerly alluded to) is north from the Royal Observatory at Paris 125522.2 toises, which are equal to 133775.3 fathoms. And from p. 51. and 57. it further appears that, by the mean of two suites of triangles, Dunkirk is east from the meridian of Paris 1420.41 toises, which are equal to 1513.8 fathoms. Again, at p. 276. of the same Book, Dunkirk is said to be north 125517 toises, and east 1430 toises, which are respectively equal to 133769.7 and 1524.2 fathoms. And, lastly, at p. 36, of the *Déscription Géométrique de la France*, of the same Author, published in 1783, and which being the latest should of course be the most correct work, Dunkirk is made north from the Royal Observatory 125495 toises, and east from its meridian only 1416 toises, which are respectively equal to 133746.3 and 1509.1 fathoms. Now, without pretending here to enter into the investigation of the various corrections + and - which have been applied to the angles of the triangles,

angles, to bring out these different results, we shall abide by the first, that being immediately produced by the mean of the observations without any arbitrary correction whatever; and being, with regard to easting, nearly a mean between the two extremes; and since we have it in our power to settle with great precision the longitude of Dunkirk, and likewise the point M, with regard to Greenwich, we shall then be enabled to determine the difference of longitude between the two Royal Observatories within a mere trifle of the truth.

By Comte DE CASSINI's triangles, executed in the autumn of 1787, and communicated in January 1789, it appears, that Hondscôte is south-eastward from the meridian of Dunkirk $67^{\circ} 53' 20''$; which angle being subtracted from the total angle between Hondscôte and Calais $144^{\circ} 53' 28''.5$, being the sum of the three angles at Dunkirk in the XLth, XLth, and XLth triangles, there remains $77^{\circ} 0' 8''.5$ for the angle that Calais is south-westward from the meridian of Dunkirk. And this last angle being again subtracted from 180° , we have $102^{\circ} 59' 51''.5$ for the angle between the same meridian produced northward, and a line drawn from Dunkirk through M to Calais.

Now the two distances 133775.3 and 1513.8 fathoms being severally reduced in the proportion of 39809.94 to 39808.7 the two lengths assignable to the base on the Strand near Dunkirk, as formerly established, we have 133771.1 fathoms for the distance in British measures of the parallel of Dunkirk from that of the Royal Observatory at Paris; and 1513.75 fathoms for the distance of Dunkirk eastward from its meridian. Again, making use of the angle $77^{\circ} 0' 8''.5$, and its complement to $90^{\circ} = 12^{\circ} 59' 51''.5$, we have 1553.56 fathoms for the direct distance between Dunkirk and the point M;

also 358.6 fathoms for the space that M is southward from Dunkirk. But the distance of Dunkirk from the Royal Observatory at Paris, given in the 276th page of M. CASSINI DE THURY'S Book, when reduced in the proportion of the two bases becomes 133765.7 fathoms, and taking a mean between this number and that formerly found 133771.1 fathoms, we have for the mean distance of Dunkirk from the Observatory 133768.4 fathoms, from which subtracting 358.6 fathoms, the mean southing of M from Dunkirk, there will ultimately remain 133409.8 fathoms for the distance between M and the Royal Observatory at Paris, measured on the meridian.

Now, since in the XLth triangle we have the distance of N.D. at Calais from Dunkirk 123734.8 feet, if from this number we subtract 1553.56 fathoms = 9321.36 feet, there will remain 114413.4 for the distance of the point M from Calais. Thus, with the supplemental angle to 360° at Calais, *viz.* $139^\circ 17' 33''.2$, contained within its now known sides, we are finally enabled to complete the XLVth triangle, and thereby to determine the situation of M with regard to Dover.

XLV.	{	N D. at Calais	.	.	.	139° 17' 33.2	} The spherical excess above 180° = 2".42.
		Dover Castle	.	.	.	18 24 37.3	
		M near Dunkirk	.	.	.	22 17 49.5	
							Feet.
	{	N.D. at Calais from	{	Dover Castle	.	.	137449.9
		M near Dunkirk		.	.	114413.4	
		Dover Castle from M	.	.	.	236273.7	
		Also, Dover is from Dunkirk	.	.	.	243291.3	

ART. III. *Result of the trigonometrical operation, in as far as relates to the geodetical situation of the different stations, with regard to the Royal Observatory at Greenwich.*

Having, by the preceding calculations of the lengths of the sides and measures of the angles of a continued series of

forty-five triangles, determined the relative situation of every station with regard to those nearest adjacent to it, we are next to shew, from these *data*, and the angles which Norwood and Severndroog Castle make with the meridian of Greenwich Observatory, the situation of each station with respect to that meridian, to its perpendicular, and also the direct or diagonal distance with the bearing from the Observatory itself. These various determinations are contained in the six first columns towards the left hand of the annexed table of results, wherein the stations are likewise distinguished into two sets, as situated to the westward or eastward of Greenwich.

By means of a scaffold, perfectly similar in principle to that formerly described, but more slight as being made for the temporary purpose only, the stand of the instrument was raised to the height of thirty-eight feet above the floor of the transit-room of the Observatory. At this elevation all the surrounding objects which we wished to observe (St. Paul's excepted, which is hidden by the camera-turret of the great room) could be distinctly seen, and the angles between them and the south meridian mark accurately measured. As that mark is but at a short distance, namely, about 1500 feet from the transit, and consequently $\frac{1}{10}$ th of an inch corresponding to about a second of an angle on the mark, it was therefore very necessary that the center of the instrument should be brought with great precision over the center of the axis of the transit-telescope underneath. In this operation, and indeed in every other while at Greenwich, the Astronomer Royal gave us his best assistance. In the first place, the central point of the axis was determined by the intersection of diagonal lines drawn across the square part in the middle. On this square part, when the telescope was in its horizontal position, a basin of quicksilver was placed,

placed, having a small cross made of two thin bits of wood fitted to the inside of the basin, and lying very near the surface of the quicksilver, in such a manner as to make the center of the cross co-incide with the intersection on the brass underneath. A small perspective glass being then fixed in a moveable board under the center of the instrument, this was made to slide at right angles to itself in the direction of the meridian and that of the axis of the transit, until the center of the cross coincided with the axis of vision in looking downwards. The board being there fastened, and the perspective removed, the intersection of silk threads stretched across the board, marked very accurately the point corresponding with the center of the transit, over which the center of our instrument was brought by the help of the plummet. The second method was still more direct. Dr. MASKELYNE had an object glass prepared for his transit telescope of a focus suited to the vertical height of the stand of our instrument above it. This glass being applied to the transit, and the aperture contracted by a piece of pasteboard with a circular hole in the middle, a very small pin-hole being likewise made in the board at top, the same was gradually moved by directions from the observer below, looking through the telescope in its vertical position, until the pin-hole nicely co-incided with the axis of vision. The instrument was then brought as before, by the help of the plummet, exactly over the pin-hole. In this manner, which was that adhered to, no doubt remained of more than about $\frac{1}{100}$ th part of an inch, with respect to the center of the instrument being in the intersection of two vertical planes passing through the axis of vision and that of motion of the transit underneath. After having remained a week, the co-incidence of the pin-hole

hole with the axis of vision of the telescope was tried, and found to have suffered no alteration.

In the VIIIth triangle, the angle at Greenwich, between Severndroog Castle on Shooter's Hill and the station on Norwood heights, hath been shewn to be $111^{\circ} 56' 50''$. By several observations on different parts of the circle, Norwood station was found to be westward from the meridian $38^{\circ} 7' 16''$, which of course leaves for the angle that Severndroog Castle is eastward from it $73^{\circ} 49' 34''$; and either of these two angles is supposed to be within a very small fraction of a second of the truth.

Now, with the sides and angles of the series of triangles already known, and the angle $38^{\circ} 7' 16''$ now given, which Norwood makes with the meridian of Greenwich towards the west, it will be sufficiently obvious to those who are in the least acquainted with plane trigonometry, that the distances of that or of any other station of the series, from the meridian of Greenwich and from its perpendicular, are easily obtained. Nevertheless, that those who are but little conversant with matters of this sort may themselves be able to examine the computations whereby the columns towards the left-hand of the annexed table have been supplied, we shall give one example, which will serve for the whole.

Suppose (Plate X. fig. 1.) GM to represent the meridian of the transit-room at Greenwich; GW the perpendicular to that meridian produced indefinitely towards the west; N the station at Norwood, and H that at Hundred Acres, whose distances are required, that is to say, westward from the meridian, and southward from the perpendicular: then through the stations N and H, let dotted lines be drawn parallel to the meridian and perpendicular respectively, whereby four parallelograms will

will be formed. In the first, or that which is nearest to Greenwich, having GN in the VIIIth triangle given = 31274.48 feet, and the angle $NGm = 38^{\circ} 7' 16''$, with its complement $NGp = 51^{\circ} 52' 44''$, it follows, that Nm representing the distance of Norwood westward from the meridian is = 19306.54 feet; and Np representing its distance southward from the perpendicular is = 24603.86 feet. Again, by attending to, and summing up the angles round the point N, we shall find the angle $GNH = 175^{\circ} 44' 36''.82$, which wanting $4^{\circ} 15' 23''.18$ of 180° , shews that the direction of the side NH inclines so much more to the westward than the angle NGm . Wherefore $NGm = 38^{\circ} 7' 16'' + 4^{\circ} 15' 23''.18 = 42^{\circ} 22' 39''.18 = HN_s$, is the angle which the line NH makes with N_s , a parallel to the meridian of Greenwich drawn through the point N. Now, in the supplemental parallelogram, having the diagonal $NH = 35648.21$ given in the VIIth triangle, and the angle $HN_s = 42^{\circ} 22' 39''.18$, also its complement = $47^{\circ} 37' 20''.82$, making use of NH as radius, and these two last angles respectively, we have $sH = 24027.36$ feet for the space that H is more westward than N; and $wH = 26334.04$ feet, that H is more southward than N. Hence $mN + sH = 43333.9$ feet is the space that H is to the westward of Greenwich; and $pN + wH = 50937.9$ feet is the space that H is southward from the perpendicular to the meridian of Greenwich. Lastly, with these two given sides, and the contained angle 90° , we find the angle $MGH = 40^{\circ} 23' 18''.54$, that Hundred Acres is south-westward from the meridian of Greenwich; whence the direct or diagonal distance $GH = 66876.73$ feet. Now, by referring to the table of results, for the two first stations westward from Greenwich, the numbers brought out in this example will be found in the left-hand columns under their respective heads, and so it would be with
the

the rest. In another place we shall have occasion to point out, how the columns towards the right-hand of the said table have been filled up.

SECTION FIFTH.

On the difference between horizontal angles on a sphere and spheroid. Plate X.

IN the Paper of 1787, various computations were given concerning the figure and dimensions of the earth, founded chiefly on the actual measurement of different arcs of the meridian in different latitudes, some of them very remote from each other. From the alternate comparison of these results it appeared, that the figure assigned to the earth by M. BOUGUER in his second spheroid agreed better with these measured portions of the curve, as so many *data*, than any of the other *hypotheses*. Hence it naturally occurred, that the trigonometrical operation which we were then about to commence might probably throw some further light on this intricate subject, which, for a great length of time, has engaged a considerable share of the attention of the scientific world. In the consideration of this matter, a new and curious point, not formerly attended to, and immediately connected with our operation, presented itself for investigation, viz. *supposing the earth to be a spheroid, such as M. BOUGUER's, considerably flattened at the poles, what might be the difference between horizontal angles observed with a fine instrument on that spheroid, and on a sphere?*

The

The following solution * of that important problem, being the only unexceptionable one that I have received, is here given in the author's (Mr. DALBY's) own words.

Let CE and CP (Plate X. fig. 2.) represent the equatorial and polar semi-diameters of the earth, considered as a spheroid flattened at the poles; PE and PN two meridians; pe and pn two corresponding ones (that is, in the same planes) on a sphere, having the same center C. Let the points a, b, A, B , on the sphere and spheroid have the same latitudes respectively. Draw the radii aC, bC , and the verticals AG, BW.

Then, because the angles AON, BDE, in the spheroid, are always equal to the latitudes of the points A, B, these angles are therefore respectively equal to the angles aCn, bCe , in the sphere, and consequently the verticals AG, BW, are parallel to the radii aC, bC .

Let the latitude of B or b be greater than that of A or a ; and let it be required to make the horizontal angle PAr on the spheroid equal to the angle pab , or what the horizontal angle would be on the sphere.

Because the angle pab is measured by the inclination of the planes, aCb, aCp , and AG is the common intersection of all the planes of the vertical circles at A, and is parallel to aC , and in the same plane; therefore, when the horizontal angle PAr is equal to the angle pab , the planes GAR, CAB, must be parallel to each other; and consequently Gr, the line where the plane GAR intersects the plane of the meridian EP, is pa-

* From my correspondence by letter, and otherwise, with Dr. MASKELYNE, I had reason to hope, that he would have favoured me with some communication on this subject. No doubt, he has been prevented by other business; but he will probably give his method of solving spheroidical triangles to the Royal Society on some future occasion.

parallel to Cb in the sphere, or WB in the spheroid. Hence, if from G , the point where the vertical AG meets the axis, we draw Gr parallel to the vertical BW , it will give the point r in the meridian EP , making the horizontal angle PAr equal to the angle pab , or what the horizontal angle would be on the sphere.

In like manner, if the angle PBv is to be made equal to the angle pba , Wv must be drawn parallel to GA , and the plane vWB will be parallel to the plane AGr ; and therefore the angles of the spheroidical triangles PAr , PvB , as measured by the inclination of the planes, are equal to each other respectively, and equal to the spherical angles of the triangle pab .

From hence it follows, that if A be the place of an instrument which measures horizontal angles in the meridian NP on a spheroid, and BT a flag-staff set perpendicular to the surface of the earth on another meridian EP , the observed horizontal angle PAB , between the meridian PA and the flag-staff BT , will be greater than it would be on a sphere (the latitudes and longitudes being the same in both) as long as the latitude of the flag-staff is greater than that of the instrument, the excess being the angle BAr ; but if the latitude of the instrument is the greatest, as suppose it was at B , and the flag-staff at A , then the observed angle PBA will be less than it would be on the sphere, the defect being the angle ABv , which, because the planes WvB , AGr , are parallel, will be the same as the excess on the other side.

If the latitudes of the points A and B are the same, the planes WvB , AGr , will co-incide, or the verticals will meet in the same point in the axis, and therefore the observed angles will be equal to each other, and the same as they would be if observed on a sphere.

Because

Because AG , vW , BW , rG , are parallel to aC , bC , the angles vWB , AGr , will be equal to the angle aCb , or arc ab , therefore the arcs vB , Ar , will each be equal to the arc ab ; that is, they are arcs of great circles of the same value, intercepted between the meridians PN , PE , at B and A .

Draw GR perpendicular to the vertical BW ; then, because BW and rG are parallel, it will also be perpendicular to rG ; and because the axis PW is the common intersection of the planes of all the meridians, and BW , rW , are in the plane of the meridian PB , therefore GR is in that plane; and because the angle WBr , made by the vertical and meridian, and the angle GRB , are right ones, therefore GR is equal to the arc Br nearly, and consequently is nearly equal to what subtends the difference of the horizontal angles on the sphere and spheroid.

And if GS be perpendicular to the vertical GA , it will be equal to the arc Av nearly, and therefore GR , GS , or the arcs Br , Av , will be as the cosines of the latitudes of B and A .

Draw AK the tangent to the meridian at A , to meet the axis CP produced; also draw AH perpendicular to the vertical AG , to meet Gr produced; through H draw KHT , and join AT . Then, because the points K , H , are in the plane of the horizon of A , the line KHT will be in that plane; and because rH and BT are in the plane of the meridian BP , therefore HT is also in the same plane, and is what subtends the angle TAH , the true difference of the horizontal angles, which, when the spheroid is given, may be determined as follows.

From the nature of the spheroid, find the length of the vertical AG ; also the points G and W , where the verticals meet the axis: then, because the angle AKG is equal to the latitude of A , and AGH is its complement, GK and AK will be given.

M 2

Let

Let a and b on the sphere have the same latitudes and difference of longitude as A and B on the spheroid, and find the angles pab , pba , and the arc ab , or angle aCb ; then because AG is given, and the angle AGH equal to the angle aCb , AH will be given; with AH and AK , and the included angle HAK (equal to the spherical angle bap) find the angle AHK , and also KH ; then, because the triangles KHG , KTW , are in the same plane (that of the meridian BP) and GH is parallel to WT , these triangles will be similar. Hence $GK : HK :: WG : TH$; now HA , HT , and the included angle AHT (the complement of AHK) being given, the angle TAH , the difference of the horizontal angles, will be given.

Example. Let the spheroid be that of M. BOUGUER; and let the latitude of A be $49^\circ 40'$, of B 50° , and their difference of longitude $0^\circ 30'$.

From the nature of the spheroid, the radii of curvature of the meridian at the equator and the pole, will be 3465507 and 3524069 fathoms nearly; their difference is 58562 fathoms, the length of the evolute of the meridian; and the vertical $AG = 3465507 + \frac{8}{15} \times 58562 + \frac{1}{5} \times 58562 \times \overline{\text{fine } 49^\circ 40'}^4 + \frac{4}{15} \times 58562 \times \overline{\text{fine } 49^\circ 40'}^2 = 3509769.5$ fathoms; also $OG = \frac{8}{15} \times 58562 + \frac{4}{15} \times 58562 \times \overline{\text{fine } 49^\circ 40'}^2 = 40307.66$ fathoms; and $DW = \frac{8}{15} \times 58562 + \frac{4}{15} \times 58562 \times \overline{\text{fine } 50^\circ}^2 = 40397.23$ fathoms. Now, the angles GOC , WDC , being = the latitudes of A and B , we get $CG = 30726.16$, and $CW = 30946.08$ fathoms, their difference being 219.92 fathoms = GW .

The sides pa , pb , being equal to $40^\circ 20'$ and 40° respectively, and the included angle = $30'$, will give the angle $pab = 43^\circ 51'.48''.2$, the angle $pba = 135^\circ 45' 16''.2$, and ab , or the angle aCb , = $27' 49''.7$.

Now,

Now, by proceeding as directed above, we get $GK = 4604232.9$, $AK = 2980006.3$, and $AH = 28412.2$ fathoms. Hence the angle $AHK = 135^\circ 45' 20''.08$, and $KH = 2979745.4$ fathoms; whence $HT = 141.37$ fathoms. This, with AH , and the included angle $AHT = 44^\circ 14' 39''.92$ (the complement of AHK) give the angle $TAH = 11' 58''.9$, the difference between the horizontal angles on the sphere and spheroid.

Hence the observed angles at A and B would be $43^\circ 51' 48''.2 + 11' 58''.9 = 44^\circ 3' 47''.1$, and $135^\circ 45' 16''.2 - 11' 58''.9 = 135^\circ 33' 17''.3$.

If the figure is an ellipsoid having the same *axes*, the angle TAH will be found $= 8' 4''.4$.

It may be remarked, that the angle TAH , or the horizontal angle TAK , diminishes or augments as the point observed in TB is elevated or depressed; this variation is however too small to be worth attending to in practice, as may be shewn in the following manner.

Let the spheroid be M. BOUGUER's (because the difference will be greater than on an ellipsoid); and let the points A, B, fig. 3. have the same latitudes and difference of longitude as above; also, let BT be the flag-staff, and through B draw GBn .

Now, if we suppose B to be in the horizontal line nearly, the horizontal angle at A, taken between the north part of the meridian AP and the flag-staff at B, will be the angle BAP , the telescope in this case being pointed to B, and the vertical plane which it would then move in is the plane $nBGA$; but if the telescope is directed to some point T in the flag-staff above B, the angle TAP in this case will evidently be less than it was in the former by the angle nAT nearly; and consequently it diminishes as the observed point T is elevated; and it is also evident,

evident, that it will be augmented as the point observed is below B.

The latitudes of A and B being $49^{\circ} 40'$ and 50° , and their difference of longitude $30'$, BG will be nearly equal to AG, or 3509769 fathoms, and GR being equal to 141.36 fathoms, and the angle GRB a right one, we have BG (3509769) : rad. :: GR (141.36) : fin. $8''$, the angle GBW, or TBn . Now, supposing the point T to be a mile above the surface, this with the angle $nBT = 8''$, will give Tn equal to about three inches; but Tn is in the plane of the meridian PBE, and consequently would be seen obliquely, if viewed from A, because the angle ABT is about 135° , and therefore Tn must subtend a very small angle at the distance of $33\frac{1}{2}$ miles, which is nearly the distance between A and B.

From the determination of the horizontal angles that would be observed at A and B (fig. 2.) on the spheroid, if AP, BP, the co-latitudes of A and B are known, and the angles ABP, BAP, are given by observation, it follows, that the greater of these observed angles must be augmented, and the lesser diminished, by the same quantity of a degree, till the sum and difference give the opposite sides AP, BP, accurately by spherical computation, and then the third angle, or difference of longitude, will be given: for the observed angles at B and A being respectively $135^{\circ} 33' 17''.3$ and $44^{\circ} 3' 47''.1$, we have sine $135^{\circ} 33' 17''.3 + 11' 58''.9$: sine AP :: sine $44^{\circ} 3' 47''.1 - 11' 58''.9$: sine BP accurately; but taking the angles that would be observed, sine $135^{\circ} 33' 17''.3$: sine AP :: sine $44^{\circ} 3' 47''.1$: sine of an arc greater than BP; and sine $44^{\circ} 3' 47''.1$: sine BP :: sine $135^{\circ} 33' 17''.3$: sine of an arc less than AP; and this will shew if the observed angles are consistent, as angles that ought

ought to be found by observation on a spheroid flattened at the poles.

Because the sum of the observed angles at A and B on the spheroid are equal to the sum that would be observed on a sphere, the latitudes and difference of longitude being the same on both, and the differences equal, therefore the sum for computation is the same for both, and the quantity of each for computation on the spheroid may be found from the following

Theorem.

In any spherical triangle BPA (fig. 4.) if two of the sides PB, PA, and the sum of the opposite angles, $PBA + PAB$, are given, it will be,

*As the tangent of half the sum of the sides,
Is to the tangent of half their difference;
So is the tangent of half the sum of the angles,
To the tangent of half their difference.*

In the spherical triangle abp (fig. 2.), as $\text{fine } bap : \text{fine } bp :: \text{fine } abp : \text{fine } ap$; that is, on the spheroid, $\text{fine } BvP : \text{fine } BP :: \text{fine } vBP : \text{fine } AP$. Now, the arc Bv being = the arc ba , considered as an arc of a great circle, it follows, that in the spheroidical triangle vBP , if vB , BP , and the included angle vPB are given, the other angles at P and v may be found by spherical computation, but not the third side vP . Suppose BP , Bv , are given, and the included angle vBP a right one; then $\text{rad.} : \text{fine } BP :: \text{cotang. } Bv : \text{cotang. angle } BPv$; therefore, if the latitude of the point B , and the angle BWv , or the quantity of the arc Bv , as an arc of a great circle perpendicular to the meridian at that point, are given on a spheroid, the difference of longitude may be found by spherical computation, but not the latitude of the point v .

But if the spheroid is known, the latitude of a given point (v) in a great circle perpendicular to the meridian, may be found nearly from what has been delivered above. Thus, as $\text{rad.} : \cosine\ BP :: \cosine\ Bv : \cosine\ \text{of an arc } (PA)\ \text{less than } Pv$, the co-latitude of v . Now, with the latitude (suppose of the point A) thus found, and the given latitude of B , find GS (fig. 2.) which will be very nearly equal to the arc Av , and the value of this, as an arc of the meridian, being added to PA , will give Pv , the co-latitude of v .

SECTION SIXTH.

Manner of determining the latitudes of the stations. Application of the pole-star observations to computations on different spheres, and also on M. BOUGUER's spheroid, for the determination of the differences of longitude. Ultimate result of the trigonometrical operation, whereby the difference of the meridians of the Royal Observatories of Greenwich and Paris is determined. Plate X.

ARTICLE I. *Preamble, shewing the general principles adopted for settling the latitudes of the stations.*

In the Paper of 1787, so often quoted, and which was intended only as a sketch of the mode then proposed to be followed in conducting the recent trigonometrical operation, we had occasion to shew, that the measured arc of the meridian between the point M near Dunkirk, and Perpignan situated at
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the bottom of the Pyrenean mountains, corresponding to an arc in the heavens of nearly $8^{\circ}\frac{1}{3}$ of latitude, differed but little from what should be its true length, supposing the earth to have the figure and dimensions assigned to it by M. BOUGUER in his second spheroid. Here, however, it is become necessary to take notice of some mistakes * that, through inadvertency, were fallen into in the computed lengths of the arcs, which, although they affect in a certain degree the accuracy of the numbers brought into comparison, do not invalidate the general reasoning there advanced, and the only thing meant to be established, namely, that M. BOUGUER's hypothesis agreed better with actual measurement on different parts of the surface of

* The mistakes adverted to in the text were of three kinds. First, an erroneous mode of summing up the lengths of the arcs from the lengths of the degrees, although these taken separately were very accurately computed: for instance, the 43d was taken as that extending from 42 to 43, whereas it should have been taken for the middle point, that is, from $42\frac{1}{2}$ to $43\frac{1}{2}$, and so on in regard to others. Hence the arcs are all made somewhat too long. The second was the omission of the value of $93\frac{1}{2}$ toises in estimating the length of the celestial arc between Greenwich and Perpignan, the sector with which the stars were observed having stood so much to the northward of the church of St. Jaumes, the point to which the triangular measurement corresponded. The third was fallen into from not knowing that the French observations of the stars had been corrected for the nutation of the earth's axis, in a Paper of M. DE LA CAILLE's, inserted in the Memoirs of the Academy of Sciences for the year 1758, whereby all the lengths of the celestial arcs were thereby in some degree changed from what had been assigned to them respectively in the Book, *La Méridienne vérifiée*, published in 1744. From the same Paper it further appears, that they rejected altogether their observations at Perpignan, as being probably affected by the attraction of the Pyrénées. With regard to that part of the Table of Comparison in the Paper of 1787, which is affected by these errors, the only thing that now can be done is to annex to this paper a corrected slip, which may be referred to occasionally, or cut off and pasted over the former.

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the earth, than any of the others with which it was compared.

In proof of this, we need only for the present remark, what will be made fully to appear hereafter, that the distance between the parallels of Greenwich and Dunkirk, or Greenwich and M, being now added (by our trigonometrical operation) to the measured length of the meridian of France, the measured and computed sections of the united meridian will be found to agree almost exactly at Paris; that the excess of the measurement is but of the value of 3'' or 4'' at Bourges; only of 6'' at Rodés; and even as low down as Perpignan, comprehending in the whole an arc of the heavens of more than $8^{\circ}\frac{3}{4}$, the excess is not greater than what would answer to between 16'' and 17'', the chief part of which is probably owing to the attraction of the plummet of the sector by the Pyrenees. In the Paper of 1787, the effect had been assumed at a quantity equal to about 10''. But every thing on this head must be considered as merely matter of supposition, which cannot be determined one way or other until triangular measurements shall have been extended beyond the Pyrenean mountains into Spain, and corresponding observations of the stars made on both sides with the same instrument, which should be one of the best that could possibly be invented for the purpose. In the mean time, since the French have rejected their own observations at Perpignan, we shall avoid drawing any conclusions with regard to latitudes from the observations to the southward, and confine ourselves to those immediately connected with our operation, made at the northern stations of the meridian.

In carrying on the trigonometrical operation, it never was proposed that we should attempt to determine the latitudes of

the stations, by actual observations of the zenith distances of stars, which, with the very best instruments hitherto used for that purpose, could not have been done nearer than about 1'' of an angle in the heavens, answering in these parts to 101 feet on the surface of the earth. Even if we could have been supplied with a sector so far surpassing the old ones (such perhaps as Mr. RAMSDEN may hereafter invent) that would have given zenith distances to one-tenth part of a second, or about ten feet on the surface of the earth, the application of it in our operation would have been mere loss of time: for the Astronomer Royal having settled the latitude of Greenwich $51^{\circ} 28' 40''$, to within less than half a second of the truth; and the geodetical situation of each station of our series being determined so accurately with regard to that point, as to leave no where an uncertainty of more than one or two feet; we have thereby been able to determine the relative latitudes to a small fraction of a second. Here, however, it is to be understood, that we have adhered to M. BOUGUER's scale, as answering almost exactly in the narrow space of $26' 51''$, or thereabout, of latitude between Greenwich and M, to which our operations have been confined.

That this mode of settling the latitudes of our stations is extremely accurate, will more fully appear from the following considerations. In the general computation of spherical triangles, a sphere whose diameter is a mean between the longest and shortest of M. BOUGUER's spheroid has been adopted, because it was obvious, that in our latitudes the degree of such a sphere could not differ sensibly from the mean degree of the spheroid. Thus the degree of the sphere 60859.1 fathoms answers (as may be perceived by consulting the table in the Paper of 1787) to the degree of the meridian on the spheroid in

the latitude of $51^{\circ} 5'$. Again, if the total length of one-fourth part of the spheroidical meridian of the earth, between the equator and the pole, 5478094.4 fathoms be divided by 90° (*Fig. de la Terre de BOUGUER*, p. 310. and 311.), we shall have 60867.72 fathoms for the mean degree of the meridian, which in the same table will be seen not to differ sensibly from that answering to the latitude of Greenwich; in or near which parallel the curves of such a sphere and M. BOUGUER's spheroid intersect each other, as will be readily conceived by referring to and considering the representation of them, in Plate X. fig. 3.

ART. II. *Of the pole-star observations in general.*

It became necessary, in the preceding article, to point out in what manner the latitudes of our stations have been deduced from their relative situation with regard to Greenwich; because the method adhered to of settling the differences of longitude by the observations of the pole-star, which could rarely be made except on one side, that is to say, at night, when the star was eastward from the pole, implied as a matter of course, that the latitude of the station should be accurately known, for the computation of the star's azimuth. With the declination of the star, settled to so great a nicety as it has been by the Astronomer Royal, and the latitude of the place given, a single azimuth was sufficient for obtaining immediately the true direction of the meridian. Much time would have been uselessly lost in attempting to get observations of the star in day-light when on the west side of the pole, whereby the double azimuth would have been obtained; and in that case the bisection of the angle would have given the true meridian of the place, without the knowledge of its latitude.

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For the purpose of the pole-star observations a small table had been previously computed, of the exact times of the star's being in the east and west; whence the moments of its greatest elongation were readily known. On these occasions the Board of Longitude's *præmium watch*, by the late Mr. HARRISON, was made use of. Its rate of going all the time that it was in the field in 1787, was very uniformly $9\frac{1}{2}$ seconds a day faster than mean time. But in the winter months the watch gradually changed its rate from *plus* to *minus*, and when it was carried into the field in 1788, and, during the five weeks that it continued there, it regularly lost on mean time from $3\frac{1}{2}$ to 4 seconds each day; having in that short interim been twice compared in Argyll-street, with an excellent clock made by CUMMING, with an improved ELLICOT's pendulum.

With regard to these pole-star observations, whereby the differences of longitude, or the angles of convergence of the meridians towards each other, have been determined, it is necessary to remark, that although some few were made to the westward of Greenwich, yet these were not at sufficient distance from it, and also taken of too short sides, to afford results that were perfectly satisfactory and conclusive. It is on the observations to the eastward only, and chiefly on those made at Goudhurst and Botley Hill, which are upwards of twenty-three miles from each other, and reciprocally visible, that we have relied for the scale of degrees of a great circle perpendicular to the meridian in these latitudes; whence those of longitude have been obtained. The observations made at Folkestone Turnpike, which is upwards of fifty-eight miles in direct distance from Greenwich, and where, fortunately, the double azimuth of the pole-star was obtained, are perfectly consistent with those taken at Goudhurst and Botley Hill. But
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when at Fairlight Down we had no observations of the star, being at that time so much engaged with the other essential business of the triangles, and particularly with the intersection of the lights on the Coast of France, as to render it impossible to attend to any thing else, even if the weather had proved less unfavourable than it was at the period alluded to, for celestial observations.

ART. III. *Pole-star observations at Goudhurst and Botley Hill applied to computations on the mean sphere.*

Let B (Plate X. fig. 5.) be Botley Hill; PBR its meridian; G Goudhurst; W Wrotham Hill; T Tenterden; RG an arc of a great circle passing through G, and falling perpendicularly on the meridian BR; also let $\ast\ast$ represent the circle of the pole-star's apparent declination; and $B\ast$, $G\ast$, be two azimuth circles touching that circle.

August 14, 1788, at Goudhurst, the angle $\ast GT$, or that between the pole-star, when at its greatest apparent distance from the pole on the east side of the meridian, and the reverberatory lamp at Tenterden was observed †, 104 32 19½

The angle BGT, between the lamp at Botley Hill and Tenterden, was repeatedly observed, 167 43 56

Their difference = angle $\ast GB$ is, 63 11 36½

† The observations of the pole-star at Goudhurst and Botley Hill were repeated for several nights at each place; but these here given are the most exact. At Goudhurst the angle which the star made with the lamp being noted, the telescope removed, and the plane of the instrument being turned 180°, or half round, the telescope replaced and directed again to the star, the difference on the circle was found to be only 1''¼. The same method was universally adhered to in all places where observations of the star were obtained. At Botley Hill, in particular, the difference between the readings was no more than 1''½.

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August 23, 1788, at Botley Hill, the angle *BW, or that between the pole-star at its greatest apparent elongation and the lamp at Wrotham Hill, was observed,

76 21 37

The angle WBG, by repeated observations, was,

40 4 42

Their sum = angle *BG is,

116 26 19

In order to obtain the star's azimuth at each place, we may take, without producing any sensible error, the latitudes of G and B, as they would be found on M. BOUGUER's figure, which we have already announced, and will hereafter prove to be consistent with observation. Thus B, or Botley Hill, is south from Greenwich $72882\frac{1}{2}$ feet, and nearly on the same meridian; wherefore its latitude will be $51^{\circ} 16' 41''.54$, and its co-latitude BP of course is $= 38^{\circ} 43' 18''.46$. Now, P* the apparent distance of the star from the pole at that time being $= 1^{\circ} 49' 22''.84$, in the right-angled spherical triangle P*B, we have $\text{fine BP} : \text{rad.} :: \text{P*} : \text{fine } 2^{\circ} 54' 54''.2$ equal to the angle *BP, the star's azimuth from the north. And this being added to the angle *BG observed $116^{\circ} 26' 19''$, we have the angle *BP $= 119^{\circ} 21' 13''.2$ for that comprehended between the meridian and Goudhurst.

The distance of Goudhurst from the perpendicular to the meridian of Greenwich is 132592 feet, and its distance from the meridian of Botley Hill, on a perpendicular to that meridian, is 106171 feet nearly = GR. Hence the latitude of the point R is $51^{\circ} 6' 52''.89$; therefore $\text{RP} = 38^{\circ} 53' 7''.11$, and $\text{RG} = 106171 \text{ feet} = 17' 19''.7$ nearly. Hence, as $\text{rad.} : \text{cofine RP} :: \text{cofine RG} : \text{fine } 51^{\circ} 6' 49''.7$, the latitude of G nearly; therefore $\text{GP} = 38^{\circ} 53' 10''.3$; and P* the star's apparent distance at the time being $1^{\circ} 49' 25''.34$, we have the angle

Aug 23

PG

PG*, the star's azimuth = $2^{\circ} 54' 20''.8$, which being subtracted from the angle BG* observed at Goudhurst between the lamp on Botley Hill and the star, there remains the angle BGP = $60^{\circ} 17' 15''.7$ comprehended at Goudhurst, between Botley Hill and the meridian.

Now with these *data* let us suppose, in the first place, the earth to be a sphere, whose diameter is a mean between the longest and shortest of M. BOUGUER's spheroids, the latitude of B, and of course its co-latitude BP, given; also the angles PBG and PGB respectively $119^{\circ} 21' 13''.2$ and $60^{\circ} 17' 15''.7$, we shall then have PG the co-latitude of G, and the angle BPG or difference of longitude of B and G. And because the degree of such a sphere contains 60859.1 fathoms, the latitude of Botley Hill will then be $51^{\circ} 16' 41''.45$, and BP its co-latitude = $38^{\circ} 43' 18''.55$. This last side, with the former angles PBG and PGB respectively $119^{\circ} 21' 13''.2$ and $60^{\circ} 17' 15''.7$, give PG = $38^{\circ} 53' 6''.72$ the co-latitude of G; and also the angle BPG, the difference of longitude of the points B and G equal to $27' 36''.7$. Again, in the right-angled spherical triangle PRG, rad. : tang. GP :: cosine of the angle RPG : tang. $38^{\circ} 53' 3''.47$ = RP. But the point R is 22094 fathoms south from Greenwich, and nearly on its meridian, therefore its latitude will be $51^{\circ} 6' 52''.8$; and hence PR the co-latitude will be $38^{\circ} 53' 7''.2$, which exceeds PR formerly found by spherical computation to be $38^{\circ} 53' 3''.47$ by $3''.73$, an arc equal to 63 fathoms. Also RG, the distance of Goudhurst from the meridian of Botley Hill, on a perpendicular to that meridian, is equal nearly to 17695 fathoms, which, allowing 60859.1 fathoms for a degree, corresponds to an arc of $17' 26''.7$. But spherical computation formerly gave RG = $17' 20''$, the difference consequently is $6''.7 = 113\frac{1}{4}$ fathoms; therefore the earth cannot be this mean sphere, which was assumed

assumed for the purpose of exemplification, because its degrees, in the direction of the meridian, differ so little in these latitudes from those of M. BOUGUER's spheroid.

ART. IV. *The same pole-star observations applied to computations on a sphere of greater dimensions.*

Let us suppose, in the second place, the earth to be a sphere of such magnitude as to have degrees of a great circle containing $61253\frac{1}{2}$ or 61254 fathoms, we shall then get the latitude of B or Botley Hill = $51^{\circ} 16' 46''$, the latitude of R = $51^{\circ} 7' 1''.2$, and $PR = 38^{\circ} 52' 58''.8$; also $RG = 17' 19''.9$. Now, $BP = 38^{\circ} 43' 13''.9$, and the observed angles will give the angle BPG, or the difference of longitude = $27' 36''.7$, the same as before, and the arc PG or co-latitude of G = $38^{\circ} 53' 2''.05$ *. This last side, with the angle $RPG = 27' 36''.7$ of the right-angled spherical triangle PRG, will give $PR = 38^{\circ} 52' 58''.8$, and $RG = 17' 19''.9$; that is to say, the observed angles PBG and PGB, at Botley Hill and Goudhurst respectively, are nearly the same as they would be found on a sphere of such magnitude as to have degrees containing $61253\frac{1}{2}$ or 61254 fathoms. But since the value of RG as an arc of a great circle was before found by the triangles BPG and RPG to be $17' 20''$, when the latitude of B was taken as belonging to a sphere whose degrees contained 60859.1 fathoms; and the same arc as now determined, viz. $17' 19''.9$, agrees very nearly

* It is evident, that as the latitude of B increases, the star's azimuth, or the angle $\angle BP$, and consequently the angle PBG, increase likewise. But at G the angle PGB is diminished by the increase of the angle $\angle GP$, or the azimuth; and therefore if the difference of the latitudes of B and G remains the same, or nearly the same, the sum of the angles PBG, PGB, will also be nearly the same; wherefore no sensible difference in the angle BPG, or difference of longitude, will be found on this account.

with the former, although the latitude of B be now taken on a sphere whose degrees contain $61253\frac{1}{2}$ fathoms, it obviously follows, from these recent observations, *that whatever the precise figure of the earth may be, or the ratio between its diameters, the degree of a great circle upon it perpendicular to the meridian, cannot in these latitudes differ much in length from $61253\frac{1}{2}$ fathoms.*

ART. V. *The pole-star observations at Folkestone Turnpike applied to computations on the same greater sphere.*

Let G (Plate X. fig. 6.) be Greenwich; PR its meridian; F, H, and T, the stations at Fairlight Down, High Nook, and Folkestone Turnpike, respectively; also let PF and PT be meridians passing through F and T; and FR and Tr great circles cutting the meridian of Greenwich PR at right angles in R and r.

At the station T, on the 7th of September, 1788, at night, the angle between the pole-star, when at its greatest apparent elongation from the pole on the east side of the meridian, and the reverberatory lamp at H, was observed, . . . 123 19 3 $\frac{1}{4}$

On the following morning, Sept. 8th, the angle between the star, when at its greatest distance on the west side, and the flag-staff at H, was observed, . . . 117 30 52 $\frac{1}{2}$

The difference or double azimuth is, . . . 5 48 10 $\frac{3}{4}$

And the half sum is, . . . 120 24 57.87

This half sum $120^{\circ} 24' 57''.87$ * is the angle PTH, or that comprehended between the meridian PT and H. The angle HTF,

* By taking the latitude of T as determined on M. BOUGUER's spheroid, = $51^{\circ} 5' 45''.3$ nearly, the co-latitude or TP is equal to $38^{\circ} 54' 14''.7$, and the star's apparent distance at the time being $1^{\circ} 48' 18''.03$, we have, as sine 38°

HTF; or that between the lamp at H and the white lights repeatedly fired at F, was twice observed $22' 48''$; therefore $120^\circ 24' 57''.87 + 22' 48'' = 120^\circ 47' 45.87$ is the angle PTF, that the station on Fairlight Down makes with the meridian of Folkestone Turnpike.

Now, rT being equal to 45827.88 fathoms, and $RF = 23884.68$ fathoms, if we take $61253\frac{1}{2}$ fathoms $= 1^\circ$, we shall have $Gr = 22871$ fathoms $= 22' 24''.18$; $GR = 36436.1$ fathoms $= 35' 41''.42$; $rT = 44' 53''.4$; and $RF = 23' 23''.75$; therefore PR , the co-latitude of R , will be $39^\circ 7' 1''.42$; and that of r or Pr will be $38^\circ 53' 44''.18$. Hence, in the right-angled spherical triangle PRF , we shall have the angle $RPF = 37' 4''.901$, and $PF = 39^\circ 7' 7''.294$. Further, the triangle PrT gives the angle $rPT = 1^\circ 11' 29''.143$, and $PT = 38^\circ 54' 5''.98$. Now, $1^\circ 11' 29''.143 - 37' 4''.901 = 34' 24''.242 =$ the angle FPT . This last angle, with the two containing sides PT and PF , give the angle $PTF = 120^\circ 47' 44''.75$, the same as it was actually observed very nearly. And hence we have another strong proof, *that on this part of our earth the degree of a great circle, perpendicular to the meridian, cannot differ much in length from $61253\frac{1}{2}$ fathoms, whatever may be its real figure, which cannot be determined until these observations shall have been compared with others that may hereafter be made in the same way, and with equal care, in latitudes remote from each other.*

$38^\circ 54' 14''.7 : \text{rad.} :: \text{fine } 1^\circ 49' 18''.03 : \text{fine } 2^\circ 54' 5''.12$ the star's azimuth. Twice this angle, or $5^\circ 48' 10''.24$, agrees very nearly with the double azimuth $5^\circ 48' 10''\frac{3}{4}$, found by the observations on the 7th and 8th of September. This near agreement, at the same time that it serves to shew the accuracy of these observations in particular, and the goodness of the mode that was adopted in general, serves also to prove, that Dr. MASKELYNE has settled the declination of the pole-star to great precision.

ART. VI. *The latitude of the point M near Dunkirk, and consequently the distance between the parallels of Greenwich and M, deduced from the same length of a degree perpendicular to the meridian. Also the comparison of its length with that of the meridional degree.*

Again, let us suppose G (Plate X. fig. 7.) to be Greenwich; Pr its meridian; M the point near Dunkirk, supposed to be in the meridian of Paris; Mr a great circle passing through that point, and falling perpendicularly on Pr. Then, if we take $61253\frac{1}{2}$ fathoms $= 1^\circ$, we shall have $rM (= 89674.7 \text{ fathoms}) = 1^\circ 27' 50''.37$, and $Gr (= 25831.43 \text{ fathoms}) = 25' 18''.17$. Hence Pr will be $38^\circ 56' 38''.17$; and therefore as rad. : cosine Pr :: cosine rM : sine $51^\circ 1' 58''.5$ the latitude of M; and $51^\circ 28' 40'' - 51^\circ 1' 58''.5 = 26' 41''.5$ is the difference of latitude between Greenwich and M, or the distance of their parallels. Now, as $3600'' : 61253\frac{1}{2} :: 26' 41''.5 (= 1601''.5) : 27249.3$ fathoms; and this being added to 133409.8 fathoms, the measured arc of the meridian between M and the Royal Observatory at Paris, we have 160659.1 fathoms for the length of the terrestrial arc of the meridian comprehended between the parallels of the two Royal Observatories nearly. But the length of the celestial arc between them being $2^\circ 38' 26''$ would, at the rate of $61253\frac{1}{2}$ fathoms to a degree, give $= 161743.3$ fathoms, which exceeds the measured arc by 1084.2 fathoms. Therefore it is sufficiently obvious, that the earth cannot be a sphere of these dimensions; but it must be an oblate spheroid, on which a degree of a great circle, perpendicular to the meridian, in this way of considering it, exceeds in length the mean degree of the meridian between Greenwich and Paris in the proportion of $61253\frac{1}{2}$ to 60842, or $411\frac{1}{2}$ fathoms.

ART. VII. *Application of the results of the pole-star observations to computations on M. BOUGUER's spheroid, for the distance of the parallels of Greenwich and M.*

Hitherto the results obtained by the geodetical measurement and pole-star observations have been applied to spherical computations on two spheres suited to the different lengths of degrees found in two opposite directions, at right angles to each other, the meridian and its perpendicular; and from these computations it has been clearly proved, that the earth cannot be either of the assumed spheres.

Let us therefore, in the next place, suppose the earth to have the figure and the dimensions of M. BOUGUER's spheroid, and by way of comparison apply the same results to computations on that figure. Thus the latitude of the point r will be found $51^{\circ} 3' 12''.09$, and the arc $rM = 1^{\circ} 27' 49''.03$. Hence, as $\text{rad.} : \cosine\ rP :: \cosine\ rM : \text{fine}\ 51^{\circ} 1' 48''.85 = \text{the latitude of M nearly}$. Now, let the points r and M be represented by B and v (Plate X. fig. 2.) then will A represent the point whose latitude is $51^{\circ} 1' 48''.85$; and by proceeding in the manner formerly directed for a spheroid, we get $GW = 15.12 \text{ fathoms} = \text{to the distance in the axis between the points where the verticals from the latitudes } 51^{\circ} 3' 12''.09 \text{ and } 51^{\circ} 1' 48''.85 \text{ meet the said axis}$. Hence, as $\text{rad.} : 15.12\ (GW) :: \cosine\ 51^{\circ} 1' 48''.85\ (\text{angle}\ SWG) : 9.509 \text{ fathoms} = GS$, or the arc Av extremely near. Now the value of Av , as an arc of the meridian, is $= 0''.56$, which being added to $38^{\circ} 58' 11''.15\ (AP)$, gives $38^{\circ} 58' 11''.71 = Pv$, the co-latitude of v ; and hence the true latitude of v , or M (fig. 7.), is $51^{\circ} 1' 48''.29$, which being subtracted from $51^{\circ} 28' 40''$, the latitude of Greenwich, there remains $26' 51''.71$ for the arc between them, or distance of their

their parallels, which on this spheroid corresponds to 27248.2 fathoms, less only by 1.1 fathom than the space found, in the last article, to answer to an arc of $26^{\circ} 41'' .5$, being the distance of the same parallels on the greater sphere.

Thus the measured length of the arc between Greenwich and M, 27248.2 fathoms, being added to the measured distance of M from the Royal Observatory at Paris, we have for the total length of the arc between Greenwich and Paris 160658 fathoms, which exceeds the computed length of the same arc on M. BOUGUER's hypothesis by no more than $7\frac{1}{2}$ fathoms.

But it hath been already shewn, that whatever the precise figure of the earth may be, a degree of a great circle upon it, perpendicular to the meridian, cannot in these latitudes differ much in length from $61253\frac{1}{2}$ fathoms, being but $16\frac{1}{2}$ fathoms less than 61270 fathoms, the length of the corresponding degree on M. BOUGUER's spheroid.

As far therefore as we are enabled to judge from the result of these observations, the earth differs but little either in its latitudinal or longitudinal dimensions from what hath been assigned to it by M. BOUGUER.

ART. VIII. *Application of the pole-star observations at Botley Hill and Goudhurst, for determining the length of the degree of a great circle, perpendicular to the meridian.*

Since M. BOUGUER's scale for the degrees of the meridian hath been found to agree almost exactly with observed latitudes in this part of the earth, let us take the latitudes of B and R (fig. 5.) as they would be found on his spheroid nearly, and apply the pole-star observations at B and G, in order to find the length of the degree of a great circle, perpendicular to the

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the meridian of Botley Hill, passing through Gaudhurst. We shall then have PB, the co-latitude of Botley Hill, $= 38^{\circ} 43' 18''.46$, and PR, the co-latitude of R, $= 38^{\circ} 53' 7''.14$. Now, if the latitudes of B and R are nearly true, it follows, *that the point G must be somewhere in the great circle RG, whatever may be its longitude.* Therefore the angle BPG, or the difference of longitude between B and G, will be found in the following manner.

Augment the observed angle PBG $= 119^{\circ} 21' 13''.2$, and diminish the observed angle PGB $= 60^{\circ} 17' 15''.7$ by the same quantity of a degree, until PR determined from the triangle BPG becomes $= 38^{\circ} 53' 7''.14$ nearly; which will be when that quantity is $9' 21''$. Thus the angles for computation will be $119^{\circ} 21' 13''.2 + 9' 21'' = 119^{\circ} 30' 34''.2$, and $60^{\circ} 17' 15''.7 - 9' 21'' = 60^{\circ} 7' 54''.7$; whence the angle RPG, or difference of longitude between B and G, will be found $= 27' 36''.75$, and the arc RG $= 17' 20''.06$ nearly $= 17695$ fathoms. And hence the degree of a great circle, perpendicular to the meridian, of this new spheroid, will, in the latitude of R, contain 61248 fathoms nearly.

This follows as a corollary from what hath been already said concerning spheroidical triangles.

But since the difference of longitude between B and G was formerly determined to be nearly the same, *viz.* $27' 36''.7$, when the observed angles at these two stations, and also the latitude of B, were supposed to be on a figure different from this new spheroid; it therefore follows, *that the difference of longitude between any two stations B and G, distant in the present case from each other twenty-three miles (and they should never be much less remote) may be found with sufficient exactness, by having the*
horizontal

horizontal angles at each station observed very accurately, and the latitude of one of the stations given nearly.

The difference of longitude between Botley Hill and Goudhurst, found as above, $27^{\circ} 36''.75$, being augmented by the value of the small arc comprehended between the meridians of Greenwich and Botley Hill $= 2''.7$, we have ultimately $27^{\circ} 39''.45$ for the longitude of Goudhurst, eastward from Greenwich.

ART. IX. *Difference between observed angles on the new spheroid and that of M. BOUGUER.*

Lastly, on the subject of these comparisons, let us see what would be the difference between the observed angles at B and G, as determined on the new spheroid and on that of M. BOUGUER?

The latitudes of B and G on M. BOUGUER'S spheroid would respectively be $51^{\circ} 16' 41''.54$ and $51^{\circ} 6' 49''.66$ nearly, and the angle BPG, or difference of longitude, would be $= 27^{\circ} 36''.18$. Now, this last angle, with the two co-latitudes PB and PG, as containing sides, and supposed to form a spherical triangle, will give the angles at B and G respectively $119^{\circ} 31' 26''.47$ and $60^{\circ} 7' 3''.18$. But the observed angles at these stations would be $119^{\circ} 21' 32''.97$ and $60^{\circ} 16' 56''.68$, the common difference between them being $9' 53''.5$, which is $32''.5$ greater than $9' 21''$, as was before determined. Hence we may conclude, that in this new spheroid, founded immediately on the recent geodetical measurements and observations of the pole-star made at Botley Hill and Goudhurst, the verticals from B and G meet the earth's axis at a less distance from each other than they would in M. BOUGUER'S spheroid. The length of the vertical is shorter as well as the radius of the parallel, whereby Goudhurst,

hurst, or the point R, is less removed from the earth's axis than it would be on the former figure; and consequently it is probable, that the spheroid is less oblate.

From the preceding determinations it is further evident, that supposing the latitudes of B and G, with the horizontal angles PBG and PGB to be given by observation, not only the difference of longitude, or the angle BPG, will be obtained, but also the arc BR of the meridian, the arc RG of a great circle perpendicular to it, and the oblique arc BG, all considered as arcs of great circles of the spheroid.

ART. X. Further illustration of the manner of settling the latitudes and longitudes of the stations comprehended in the general table of results.

Having shewn, in the preceding part of this section, how the length of the degree of a great circle, perpendicular to the meridian, and also the differences of latitude and longitude, have been obtained by very accurate observations of the pole-star made at certain stations to the eastward of Greenwich, whereby we have been furnished with a scale for settling the longitudes of all the other stations where no observations of the pole-star could be had, or only such as were not to be depended upon; we shall, by way of further illustration of this matter, give another example of the calculations for the point M near Dunkirk, which will suffice for all the other stations comprehended in the general table of results placed at the end of this section, where the respective columns have been filled up by the same or a similar mode of computation.

Let G (Plate X. fig. 8.) be Greenwich; GR its meridian; Gg the perpendicular to that meridian, produced eastward; MR a parallel to that perpendicular drawn through the point

P
M;

M; and let Mg be a portion of a small circle of the spheroid, or parallel to the meridian of Greenwich, produced from M northward, until it intersects the perpendicular in the point g. Also, let MP represent the meridian of the Royal Observatory at Paris, passing through the point M, and intersecting the parallel of Greenwich in P. Further, let C represent the church of *Notre Dame* at Calais, and making, as appears by the triangles, an angle RMC of $14^{\circ} 51' 3''.9$ with the parallel to the perpendicular of the meridian of Greenwich drawn through the point M.

From the annexed general table of the results of the triangles, it appears, that $MR = gG$ contains 538048 feet = 89674.7 fathoms; and that $GR = gM$ contains 154938 feet = 25823 fathoms. Now, since great circles, perpendicular to any meridian of the spheroid, converge towards each other, as they depart from that meridian, in the same manner as the meridians themselves do in departing from the equator, but by a slower rate, it is obvious, that the perpendicular to the meridian of Greenwich, passing through the point M, must fall below or to the southward of R on that meridian, so as that $Gr : GR :: \text{rad.} : \cosine MR = 1^{\circ} 27' 51''$, considered as a portion of a great circle of the spheroid, perpendicular to the meridian of Greenwich. Hence, Gr will contain 25831.43 fathoms = $25^{\circ} 27''.9$ of latitude, and therefore the latitude of r will be $51^{\circ} 3' 12''.1$, and its co-latitude $38^{\circ} 56' 47''.9$. Also, Rr measures 8.43 fathoms, and subtends an angle $RMr = 19''.42$.

In the right-angled spherical triangle, pole rM , right-angled at r , making use of the half sum and half difference of the containing sides, r pole and rM , with the co-tan-

gent

gent of 45° we have the angle of longitude r
 pole M, $\quad \quad \quad = 2 \ 19 \ 42.5$
 The angle, pole Mr, $\quad \quad \quad = 88 \ 11 \ 21.4$

And the complement of this last, or the convergence of the meridian of M (supposed to coincide with that of Paris) to the meridian of Greenwich, $\quad \quad \quad = 1 \ 48 \ 38.6$

Also, as $\sin 88^\circ 11' 21''.4 : \sin 38^\circ 56' 47''.9 :: \text{rad.} : \sin 38^\circ 58' 11''.2$ the co-latitude of M; whence its latitude becomes $51^\circ 1' 48''.8$, from which, deducting the spheroidical correction $0''.5$, we have the true latitude of $M = 51^\circ 1' 48''.3$. The difference between this and the latitude of $r = 51^\circ 3' 12''.1$ will be $1' 23''.8$, answering on the spheroid to 1416.77 fathoms; and this last number being added to the value of the arc $Gr = 25831.43$ fathoms, we have 27248.2 fathoms for MP, or the distance of the parallels of Greenwich and M on this new spheroid.

Lastly, if to this mean distance of the parallel of M from Greenwich, we add the mean distance of the parallel of M from Paris $= 133398.8$ fathoms, we shall then have 160647 fathoms for the mean distance of the parallels of Greenwich and Paris, answering to the celestial arc of $2^\circ 38' 26''$. Hence the mean degree of the meridian between Greenwich and Paris corresponding to the latitude of $50^\circ 9' \frac{1}{2}$, contains 60838.3 fathoms, or about $1 \frac{1}{2}$ fathom less than M. BOUGUER's degree for the same latitude.

ART. XI. *Comparison of the angle between the meridian of the point M and a line drawn from thence to Calais, as approximately deduced from the British and French observations.*

In the spheroidical quadrilateral GgrM (fig. 8.), formed by three arcs of three great circles, and one of a small circle of the spheroid, we have two right angles at G and r, and two others at g and M, each greater than a right angle by $9''.7$; therefore the angle RMC, resulting from the triangles $= 14^\circ 51' 3''.9 - \text{RM}r (19''.42) = 14^\circ 50' 44''.5 + 90^\circ 0' 9''.7 (\text{CM}g) = 104^\circ 50' 54''.2$, is the angle gMC, or that which Calais makes with a parallel to the meridian of Greenwich drawn through the point M. From this last angle subtracting the angle PMg $= 1^\circ 48' 38''.6$, or the quantity by which the meridian of M (supposed to co-incide with that of Paris) converges towards that of Greenwich, there remains the angle PMC $= 103^\circ 2' 15''.6$ for the angle that the meridian of M should make with a line drawn from D, or Dunkirk, through that point to Calais, according to the British observations.

By the late French operations, the meridian of Dunkirk makes, with a line drawn through M to Calais, an angle of $102^\circ 59' 51''.5$. The convergence of the meridian of M to that of Dunkirk, on a difference of longitude of $2' 21''.54$, is $1' 49''.94$, which being added to $102^\circ 59' 51''.5$, we have $103^\circ 1' 41''.44$ for the angle that the meridian of M, or of Paris, makes with a line drawn from Dunkirk through that point to Calais. The difference between the two results $34''.16$ is nearly equal to the mean of two extremes $\left(\frac{1' 15''}{2}\right) = 37''\frac{1}{2}$, the apparent uncertainty, in the determination of that angle by

by two sets of angles given in the *Méridienne vérifiée*, as adverted to in the Paper of 1787, Phil. Transf. Vol. LXXVII. p. 195, 196.

ART. XII. *The longitudes of Dunkirk and Paris, eastward from Greenwich, determined by the sum of four differences of meridians.*

In fig. 9. let PA be the meridian of Greenwich; G Goudhurst, PR its meridian; T the station at Folkestone Turnpike, PS its meridian; C Calais, PC its meridian; D Dunkirk, and PB its meridian. Also, let AG, RT, SC, and BC, be arcs of great circles, making the angles PAG, PRT, PSC, and PBC, right ones.

The angle at Goudhurst, between its meridian and Tenterden, is $107^{\circ} 26' 40''.3$; hence, by drawing parallels to this meridian through Tenterden and the station at Allington Knoll (see the plan of the triangles) we shall get 946.6 fathoms for what the station at the Turnpike is southward, and 28098.8 fathoms for what it is eastward from the meridian of Goudhurst. Now, 60859.4 fathoms being nearly $= 1^{\circ}$ of the meridian in the latitude of Goudhurst, we have 946.6 fathoms $= 56''$ nearly $=$ the arc GR; and the latitude of Goudhurst being $51^{\circ} 6' 49''.6$, that of the point R is $51^{\circ} 5' 53''.6$; hence the co-latitude RP $= 38^{\circ} 54' 6''.4$: and since the degree of a great circle, perpendicular to the meridian, in this latitude has been shewn to contain 61248 fathoms nearly; therefore RT $= 28098.8$ fathoms will be $27' 31''.6$. This arc and RP give the angle RPS $= 0^{\circ} 43' 49''.86$ for the difference between the meridians of Goudhurst and Folkestone Turnpike.

The angle at Folkestone Turnpike between its meridian and Dover was observed $66^{\circ} 48' 35''$, and if we draw a parallel to this meridian through Dover, we shall find, that Calais is

25284.2

25284.2 fathoms eastward from the meridian of the Turnpike. Now, the latitude of Calais being $50^{\circ} 57' 30''$ nearly (which is accurate enough for computation) the length of the degree of a great circle, perpendicular to the meridian in that latitude, will be 61246 fathoms nearly. Hence, 25284.2 fathoms = $24' 46''.8$ = the arc CS; this, with the co-latitude CP ($39^{\circ} 2' 30''$), give the angle CPS = $39' 19''.48$, for the difference of longitude between the Turnpike and Calais.

By Comte DE CASSINI's Paper, communicated in January 1789, it appears, that the angle at Dunkirk, between its meridian and Broulezele, is $10^{\circ} 18' 25''$; and that between Broulezele and Calais $66^{\circ} 41' 46''\frac{1}{2}$, the sum is $77^{\circ} 0' 11''\frac{1}{2}$ for the angle at Dunkirk, between its meridian and Calais. In the same Paper we have 19349.34 toises for the distance of Dunkirk from Calais; this, with the angle $77^{\circ} 0' 11''\frac{1}{2}$, give 18853.7 toises or 20093.3 fathoms for the distance of Calais westward from the meridian of Dunkirk, which, by taking 61246 fathoms $= 1^{\circ}$ (that of a great circle perpendicular to the meridian in the latitude of Calais), is equal to $19' 41''.1$ $=$ the arc BC; and this arc, with CP the co-latitude of Calais, give the angle CPB $= 31' 15''.11$ for the difference between the meridians of Calais and Dunkirk.

The angle APR, or the difference of the meridians of Greenwich and Goudhurst, has already been found (see the end of the 8th article, and also the table of general results), $= 27^{\circ} 39.45'$

The angles	{	RPS	= 0 43 49.86
		SPC	= 0 39 19.48
		CPB	= 0 31 15.11

It hath been already remarked, that, from p. 276. *Méridienne vérifiée*, Dunkirk is 1430 toises eastward from the meridian of Paris; and that in p. 36. of the *Déscription Géométrique de la France*, we find it only 1416 toises. Now, these will give $2' 22''.6$ and $2' 21''.2$ respectively for the difference of meridians of Dunkirk and Paris; the mean is $2' 21''.9$ for the longitude of Dunkirk east from Paris; therefore $2^{\circ} 22' 3''.9 - 2' 21''.9 = 2^{\circ} 19' 42''$, or $9' 18''.8$ in time, will be the longitude of Paris east from Greenwich nearly.

Again, in fig. 10. let P be the pole; G Greenwich, PW its meridian; RD an arc of a great circle making the angle at R a right one, and passing through D; and DW an arc of the parallel of latitude of Dunkirk.

By p. 240. of the *Mem. de l'Acad.* 1758, the celestial arc between Paris and the station of the sector near Dunkirk is $2^{\circ} 11' 50''$; to which adding $5''.3$ ($= 84\frac{1}{4}$ toises) for what the tower is north from the station, we have $2^{\circ} 11' 55''.3$ for the arc between Paris and Dunkirk; therefore, if the latitude of Paris is $48^{\circ} 50' 14''$, that of Dunkirk will be $51^{\circ} 2' 9''.3$, whence its co-latitude becomes $38^{\circ} 57' 50''.7 = DP$.

From what has been said concerning spheroidical triangles, it follows, by way of corollary, that to find RD by spherical computation, when DP and the angle at P are given, it is necessary to diminish DP by a certain quantity determinable from the nature of the spheroid; this quantity is about $0''.5$ when the spheroid is M. BOUGUER's; therefore DP may be taken $= 38^{\circ} 57' 50''.2$, which is sufficiently accurate for computation.

Hence, as $\text{rad.} : \text{fine } DP :: \text{fine } 2^{\circ} 22' 3''.9 = WPD : \text{fine } 1^{\circ} 29' 19''.17 = \text{the arc DR}$. Now, 61247 fathoms being equal to 1° of a great circle perpendicular to the meridian in
the

the latitude of Dunkirk nearly, we have, as $1^{\circ} : 61247 :: 1^{\circ} 29' 19''.17 : 91175.8$ fathoms = the arc DR.

But the length of this arc DR has been found nearly the same, that is, 91176.3 fathoms (see the table of general results) by continually drawing parallels to the meridian of Greenwich through the different stations between it and Dunkirk; therefore, although that method in general is not strictly accurate, having a tendency to give the results in excess; yet it is evident, that the length of the arc of a great circle so determined will differ very little from the truth, when the series of triangles employed for that purpose are contiguous to it, and follow its direction nearly.

ART. XIII. *For the distance between the parallels of latitude of Greenwich and Paris.*

The distance between the parallels of Greenwich and Paris has already been determined in Art. X. of this section, by taking M (1420.41 toises westward from Dunkirk) as the intermediate point. Let us next see what will be the result when Dunkirk is made use of instead of M?

In fig. 10. as $\cos. RD : \text{rad.} :: \cos. DP : \cos. PR = 38^{\circ} 56' 24''.07$; but DP by observation is $= 38^{\circ} 57' 50''.7 = PW$; hence $PW - PR = 38^{\circ} 57' 50''.7 - 38^{\circ} 56' 24''.07 = 1' 26''.63 = RW = 1464.5$ fathoms nearly, by taking 60858 fathoms for a degree of the meridian, that being nearly its value in the latitude of Dunkirk.

By our operation Dunkirk is 25425 fathoms southward from Greenwich; but the great circle DR meets the meridian of Greenwich about $8\frac{1}{2}$ fathoms further south, that is to say, GR is $25425 + 8.5 = 25433.5$ fathoms, which being added to 1464.5 gives

gives 26898 for the distance between the parallels of latitude of Greenwich and Dunkirk.

Because Dunkirk is situated near the meridian of Paris, the distance between the parallels of latitude of these places will be nearly equal to what Dunkirk is north from Paris, namely, $125517\frac{1}{2}$, or 125495 toises (see the pages formerly quoted). These numbers give respectively $133770 + 26898 = 160668$, and $133746.3 + 26898 = 160644.3$ fathoms for the distances between the parallels of Greenwich and Paris, a mean between which will nearly be 160656 fathoms.

If therefore the celestial arc of the meridian between Greenwich and Paris is $2^{\circ} 38' 26''$, we get $60846\frac{1}{4}$ or $60837\frac{1}{2}$ fathoms for a degree of the meridian in latitude $50^{\circ} 9'\frac{1}{2}$, the middle point between Greenwich and Paris; and a mean of these two results $60841\frac{1}{4}$ only exceeds M. BOUGUER's degree for the same latitude about $1\frac{1}{4}$ fathom, a quantity not differing sensibly from the defect that was brought out by the computation in Art. X. Finally, therefore, by taking a mean between this and the former length $60838\frac{1}{4}$, we shall have 60840 fathoms for the degree of the meridian in latitude $50^{\circ} 9' 27''$, agreeing almost exactly with that of M. BOUGUER.

ART. XIV. *Comparison of the length of a degree of a great circle, perpendicular to the meridian in Kent, with that in the South of France.*

M. CASSINI DE THURY, in his Book *La Méridienne vérifiée*, has given us the detail of an operation carried on in the South of France in latitude $43^{\circ} 32'$, for the determination of the length of a degree of longitude, by marking, at the extremities of a long and well ascertained distance, the instantaneous explosion of gunpowder in the open air. For this purpose a

Q

series

series of triangles was extended along the shore of the Mediterranean Sea, between Cette and St. Victoire, the extreme stations from whence the light was repeatedly observed, as fired at the church of St. Maries, nearly in a central situation, at the mouth of the lesser branch of the river Rhone.

From the result of this operation, the best of the kind that has ever been executed in any country, it appears, that a degree of longitude in that latitude measures 44355.7 fathoms; whence it follows, that the degree of a great circle, perpendicular to the meridian there, must contain 61182½ fathoms, being 65½ fathoms less than the degree in the middle of Kent, latitude 51° 6' 50".

Now, if we compare this difference with that found between the corresponding degrees of great circles for the same latitudes on M. BOUGUER's hypothesis, we shall find them perfectly consistent with each other in their rate of diminution: for, by consulting the table, it will be seen, that this degree in latitude 51° 6' 50" exceeds that in latitude 43° 32' by 64.7 fathoms, agreeing within less than a fathom with the former difference.

On due consideration of so many corroborating circumstances as have been adduced in the course of this section, there seems, therefore, to be sufficient room to conclude, that the earth differs but little either in its figure or dimensions from what hath been assigned to it by M. BOUGUER. It is true, indeed, that a new spheroid has been here presented, somewhat less flat than the former, founded immediately on the British observations; and these being again compared with the result of the above-mentioned operation, whereby the degree of longitude in the South of France was determined, it is from the combination of both results that the annexed table of the

lengths of degrees of great circles and of longitude has been constructed for middle latitudes only, extending from 42° to 52° . Without the help of such a table, the new longitudes of some intermediate places, which we shall have occasion hereafter to compare with the old, could not have been so accurately computed as was wished. Now, although it is believed, that this table will be found to answer nearly in that zone of the earth for which it is intended; yet it is only offered for temporary use, until future observations of the polar star in the same parallel, but on longer distances than our recent series of triangles afforded, or the extension of operations of the same nature with ours into remoter latitudes, shall have furnished *data* for one more correct.

Table of the degrees of great circles and of longitude for middle latitudes.

Places.	Latitudes.	Deg. of great circles, perp. to the meridian in fath.	Degrees of longitude in fathoms.
	^o / "	Diff.	Diff.
South of France	42 0 0	61170.5	45458.5
	43 0 0	61178. }	44742.8
	43 32 0	61182.5	44355.7
	44 0 0	61186.5	44013.9
	45 0 0	61195. }	43271.4
	46 0 0	61203. }	42515.2
Paris Royal Observatory	47 0 0	61211.5	41746.1
	48 0 0	61220. }	40964.2
	48 50 14	61227.5	40303.2
	49 0 0	61229. }	40169.8
	50 0 0	61237.5	39362.8
	51 0 0	61246.5	38542.7
M near Dunkirk	51 1 48.3	61246.7	38518.8
Dunkirk	51 2 9.3	61246.75	38514.1
Middle of Kent	51 6 49.6	61248. }	38450.0
Greenwich Royal Observ.	51 28 40	61251.1	38148.7
	52 0 0	61255. }	37712.3

ART. XV. *Comparison of the old longitudes of some places on the skirts of the kingdom of France with what they will be when computed by the new data.*

If the preceding determinations of the longitudes of the several stations between Greenwich and Dunkirk are accurate, or nearly so, as founded immediately on the British observations, and ultimately combined with the result of the operation in latitude $43^{\circ} 32'$, it follows, that all the longitudes of the great map of France, the labour of more than half a century, will be considerably affected thereby, in proportion to the distances of the places, eastward or westward, from the meridian of the Royal Observatory at Paris respectively.

To shew the effect produced by the new *data*, we shall collect, in the following table, the latitudes and old longitudes of a few noted places on the skirts of that great kingdom, and annex to them the new longitudes resulting from computations made with new lengths of degrees of great circles, perpendicular to the meridian, corresponding to their latitudes respectively. It will readily be conceived, that the object here in view is solely this; namely, that astronomers who live near those places, and who have their time, that is to say, the directions of their meridians very accurately ascertained, may, by their future corresponding observations (which should only be occultations of the fixed stars behind the moon's dark limb) compare the old with the new longitude, and thus be enabled to satisfy the curious world, which of the two comes nearest to the truth.

Com-

Comparative table of the old and new longitudes of some noted places on the skirts of the kingdom of France.

Places.	Latitudes.	Longitudes,		Diff. of old and new long.	
		Old.	New.	in deg ^s &c.	in time.
					Sec. Thirds.
East from Paris.					
Pilier de Cette	43 24 6	1 21 7	1 20 37	0 23	1 32
Tour de Planier, near Marseilles	43 11 58	2 54 8	2 53 6	1 2	4 8
Signal of St. Victoire	43 31 52	3 15 8	3 13 58	1 10	4 40
Straßbourg (Conn. de T. 1788)	48 34 35	5 26 18	5 24 6	2 12	8 48
Ditto (Déscrip. Géomét. 1783, p. 171.)	48 34 50	5 25 0	5 23 33	1 27	5 48
West from Paris.					
Tour de Cordouan at the mouth of the Garonne, Conn. de T. 1788	45 35 15	3 30 38	3 29 18	1 20	5 20
St. Malo	43 39 0	4 22 22	4 20 37	1 45	7 0
Fort du Pilier, at the mouth of the Loire	47 2 29	4 42 20	4 40 30	1 50	7 20
Ushant Light-house	48 28 30	7 24 33	7 21 41	2 52	11 28
The greatest difference between Straßbourg and Ushant				5 4	20 16
The least difference				4 19	17 36
The mean difference				4 41½	18 46

With regard to the longitudes in the preceding table, it is only necessary to observe, that the two books of 1744 and 1783, so often quoted, are not always consistent with each other, and both do sometimes disagree with what has been placed on the margin of the map of France. It would seem, that in the *Déscription Géométrique* a scale for degrees of longitude has been used considerably greater than that corresponding with the spherical hypothesis adhered to in the construction of the map, yet still too small for what we have found to be their measure in Kent, or that resulting from their own operations in the South of France. But if a similar mode to that which they practised with so much success in the South had

had been employed in the North of France, the same sort of result as we have obtained in Kent would probably have been the consequence; in which case it cannot be doubted, that the spherical hypothesis would have been entirely rejected, and their lengths of degrees of longitude would have been suited to an oblate spheroid, whose degrees of the meridian and of great circles perpendicular to it had nearly the proportion to each other of 60840 to 61239 for the middle latitude between Greenwich and Paris, being an excess of 399 fathoms on each degree in the longitudinal direction.

On the whole, therefore, as matters stand at present, it is sufficiently obvious, that, in the total extent of the kingdom of France from Strasbourg on the east to Ushant on the west, the difference between the old and new longitude amounts to between 17 and 20 seconds of time; that is to say, the real difference between the meridians of those places, it is presumed, will not be found by future observations made on the occultations of the fixed stars, to be so great as it was formerly supposed to be by that quantity, or something approaching it very nearly.

ART. XVI. *The observations of eclipses cannot be depended upon for determining with sufficient accuracy the difference of longitude in vicinal situations.*

Finally, with regard to differences of longitude, it may not be improper in this place to remark, that, in vicinal situations, such as Greenwich and Paris, the eclipses of the sun and moon and Jupiter's satellites do not, in general*, give results

* The result deduced by the Professor PIAZZI, of the University of Palermo, from the observations of the eclipse of the sun on the 3d of June, 1788, made at Greenwich, in company with Dr. MASKELYNE and M. D'ARQUIER, as given in the Phil. Trans. for 1789, p. 58. is an exception well worthy of notice.

sufficiently near the truth to deserve even the name of an approximation. This will incontestably appear by comparing the astronomical result produced in that way with ours obtained by actual measurement on the surface of the earth, and angular observations of the pole-star. Thus, by taking a mean of a multitude of the best of these observations of eclipses, &c. collected and corrected with great care for the purpose, the difference in time between Greenwich and Paris amounts to $9' 30''\frac{1}{2}$ *, instead of being only $9' 19''$ nearly, which our operation makes it. Now, if the difference in time between these two Royal Observatories was really so much, the degree of a great circle perpendicular to the meridian in these latitudes ($51^{\circ} 6' 50''$ that of Goudhurst, or $51^{\circ} 1' 48''$ that of the point M, it matters little which of the two is taken) would be between 1200 and 1300 fathoms shorter than the degree of the meridian in the same latitudes. Hence the earth, instead of being an oblate spheroid considerably flattened at the poles, would be one extremely prolate, in proportion with regard to the former figure of more than three to one, or between 1200 and 1300 — to about 400 +.

In remote situations, such as Europe and America, Europe and the eastern parts of Asia, separated from each other by wide oceans, the differences of longitude can only be obtained by means of astronomical observations. And as these will always be liable to some error, which may be as great on a difference of one or two, or a few degrees, as on the whole 180° , it is sufficiently obvious that, to render the effect of such

* From Dr. MASKELYNE's Paper of 1787, Phil. Transf. p. 183. it appears, that the eclipses of the 1st satellite of Jupiter give immediately for the difference of meridians of the two Observatories $9' 30''\frac{1}{2}$, without being combined with observations made in other parts of Paris.

error as small as possible, occultations of fixed stars should only be made use of, for obtaining conclusive determinations.

In vicinal situations, the next best mode to angular measurement is no doubt that of marking, by means of well-regulated clocks, as was done in the South of France, the repeated instantaneous explosion of light, observed at stations as far distant to the eastward and westward of the place of explosion as the circumstances will permit in practice, these distances having been for the purpose accurately settled by trigonometrical operation. The preferable stations for experiments of this sort will be pointed out in the conclusion to the present Memoir.

TABLE containing the GENERAL RESULTS of the TRIGONOMETRICAL OPERATION.

Stations.		By Plane Trigonometry.					Longitude.		Vertical Heights.		
		Distances in Feet.		Bearing or Angle with the Meridian.	Direct Dist. from Greenwich.	Latitude.	in Degrees, &c.	in Time.	Ground above the Sea.	Telescope above Ground.	Total.
		from the Merid. of Greenwich.	from the Perp. to the Merid.								
				° ' "	Feet.	° ' "	° ' "	m. s. th.	Feet.	Feet.	Feet.
West from Greenwich	Greenwich R. Ob. Transit Room					51.28.40			170.5	43.5	214
	Norwood	19306.54	24603.86	38. 7.16 SW	31274.48	51.24.37.34	0. 5. 2.4	0.20. 9.6	380.3	9.2	389.5
	Hundred Acres	43333.9	50937.9	40.23.18.5 SW	66876.73	51.20.17.3	0.11.19.5	0.45.18.	433.8	9.2	443
	Hanger Hill Tower	67740.69	16729.21	76. 7.40.2 NW	69775.8	51.31.24.16	0.17.46.5	1.11. 6.	213.	38.	251
	Hampton Poor-house.	83086.16	18537.98	77.25.20.3 SW	85129.1	51.25.35.2	0.21.45.3	1.27. 1.2	63.5	37.5	101
	King's Arbour	102264.55	1037.69	89.25. 7.7 NW	102269.8	51.28.47.16	0.26.48.5	1.47.14.	94.8	37.5	132.3
	St. Ann's Hill	119404.04	28852.77	76.24.55.7 SW	122840.6	51.23.51.4	0.31.14.7	2. 4.58.8	321.3	20.7	342
	Wardrobe Tower of Windsor Castle	137050.54	2562.72	88.55.43.5 NW	137074.5	51.28.59.7	0.35.55.8	2.23.43.2			
East from Greenwich	Botley Hill	171.6	72882.5	0. 8. 5.6 SE	72882.7	51.16.41.54	0. 0. 2.7	0. 0.10.8	859.3	20.7	880
	Severndroog Castle on Shooter's Hill	14032.3	4069.8	73.49.34 SE	14610.58	51.27.59.8	0. 3.40.65	0.14.42.6	417.8	64.2	482
	Frant	62342.5	138460.4	24.14.23.6 SE	151848.2	51. 5.53.9	0.16.12.5	1. 4.50.	604.1	54.9	659
	Wrotham Hill	71850.3	59305.8	50.27.48.5 SE	93164.6	51.18.53.86	0.19.12.3	1.16.49.2	761.8	9.2	771
	Goudhurst	106342.5	132592.	38.43.50.1 SE	169968.8	51. 6.49.62	0.27.39.45	1.50.37.8	437.9	59.1	497
	Fairlight Down	143308.1	218611.6	33.14.46.8 SE	261396.7	50.52.38.84	0.37. 5.	2.28.20.	539.5	5.5	599
	Hollingborn Hill	151078.1	77077.6	62.58.12.2 SE	169604.1	51.15.53.5	0.39.25.2	2.37.40.8	611	5.5	616.5
	Tenterden	158317.6	148567.	46.49.11.4 SE	217109.7	51. 4. 8.15	0.41. 8.15	2.44.32.6	220.6	101.7	322.3
	Ruckinge	204801.6	149410.3	53.53.16.3 SE	253509.6	51. 3.54.9	0.53.12.6	3.32.50.4	31.9	5.5	37.4
	Lydd	209339.3	190695.6	47.40.6 SE	283174.5	50.57. 7.4	0.54.15.4	3.37. 1.6	31.7	98.7	130.4
	Allington Knoll	219926.3	144032.3	56.46.43.5 SE	262893.3	51. 4.46.1	0.57. 9.4	3.48.37.6	323.5	5.5	329
	High Nook near Dymchurch	228246.4	165672.9	54. 1.33.4 SE	282035.3	51. 1.11.7	0.59.14.6	3.56.58.4	22.1	5.5	27.6
	Padlesworth	261707.6	130836.4	63.26.16.8 SE	292590.2	51. 6.50.36	1. 8. 9.	4.32.36.	621.3	20.7	642
	Swingfield	273722.7	118730.9	66.33.2 SE	298364.1	51. 8.47.8	1.11.14.6	4.44.58.4	473.1	56.9	530
	Folkstone Turnpike	274967.3	137214.2	63.28.47.6 SE	307302.3	51. 5.45.3	1.11.29.3	4.45.57.2	569.8	5.5	575.3
	Dover Castle, north Turret of the Keep	303766.8	124319.1	67.44.34 SE	328222	51. 7.47.7	1.19. 2.1	5.16. 8.4	373.9	95.1	469
	Montlambert near Boulogne	382889.6	273450.3	54.27.59.4 SE	470509.8	50.43. 2.3	1.38.45.	6.35. 0.	639.5		
	Blancnez	394891.7	197153.5	63.28.8 SE	441371.7	50.55.31.3	1.42 17.9	6.49.11.6	444.5		
	N. D. at Calais	427456.6	184263.3	66.40.50.2 SE	465480.5	50.57.30.67	1.50.48.8	7.23.15.8	140.5		
	Point M near Dunkirk, Merid. R. Ob. Paris	538048.2	154938.2	73.56. 7.2 SE	559912.8	51. 1.48.3	2.19.42.5	9.18.50.			
	Dunkirk	547058.1	152549.1	74.25. 7.2 SE	567929.4	51. 2. 9.3	2.22. 4.	9.28.16.			

SECTION.

[To follow the TABLE of GENERAL RESULTS, page 124.

Observed Celestial Arc of the Meridian between Greenwich and Perpignan.				Computed Terrestrial Arc of the Meridian between Greenwich and Perpignan.																				
			Meas ^d Terrest ^l Arc.	on a Sphere.		on different Ellipsoïds.														on different Spheroids.				
	Latitudes.	Differences.		Semidiam.	3486975.	1 st R.S.D.	{ 179.047. to 178.047	2 ^d R.S.D.	{ 192.483 to 191.483	3 ^d R.S.D.	{ 216.06 to 215.06	4 th R.S.D.	{ 222.55 to 221.55	5 th R. S. D.	{ 230. to 229.	6 th R.S.D.	{ 310.3 to 309.3	7 th R. S. D.	{ 540. to 539.	1 st R. S. D.	{ 222.55 to 221.55	2 ^d R. S. D.	{ 179.4 to 178.4	
1	Greenwich Rl. Ob.	51. 28. 40.	0. 26. 30.7	Fath ^s	Fath ^s	Fath ^s		Fath ^s		Fath ^s		Fath ^s		Fath ^s		Fath ^s		Fath ^s		Fath ^s		Fath ^s		
	Tower of Dunkirk	-	-	26898	26891	- 7	27000	+ 102	26995	+ 97	26953	+ 55	26946	+ 48	26893	- 5	26883	- 15	26872	- 26	26946	+ 48	26893	- 5
2	Tower of Dunkirk	51. 2. 9.3	2. 11. 55.3	133758	133811	+ 53	134299	+ 541	134274	+ 516	134073	+ 315	134042	+ 284	133782	+ 24	133739	- 19	133695	- 63	134047	+ 289	133758	0
	Paris Rl. Ob.	-	-	106647	106696	+ 49	107024	+ 377	107006	+ 359	106847	+ 200	106831	+ 184	106625	- 22	106604	- 43	106584	- 63	106831	+ 184	106585	- 62
3	Tower of Bourges	48. 50. 14.	1. 45. 11.4	166115	166352	+ 237	166753	+ 638	166727	+ 612	166516	+ 401	166474	+ 359	166156	+ 41	166144	+ 29	166140	+ 25	166489	+ 374	166062	- 62
	Tower of Bourges	47. 5. 2.6	2. 44. 0.2	100448	100526	+ 78	100704	+ 256	100689	+ 241	100567	+ 119	100548	+ 100	100358	- 90	100364	- 84	100377	- 71	100547	+ 99	100287	- 161
4	N. D. at Rodés	-	-																					
5	N. D. at Rodés	44. 21. 2.4	1. 39. 6.4																					
	St. Jaumes at Perpignan	42. 41. 56.	8. 46. 44.	533866	534276	+ 410	535780	+ 1914	535691	+ 1825	534956	+ 1090	534841	+ 975	533814	- 52	533734	- 132	532668	- 195	534860	+ 994	533585	- 281
Greenwich & Perpignan																								

The measured Arc between Paris and Dunkirk is taken a Mean between 125517½ and 125495 Toises, or 133770 and 133746 Fathoms. See Sect. VI. Art. xii.
The Latitudes of Dunkirk, Bourges, Rodés, and Perpignan, are deduced from the Celestial Arcs given by M. DE LA CAILLE, in pag. 240, 241, *Mem. de l'Acad. Mém. vérifiée.*

The Latitudes of Dunkirk, Bourges, Rodés, and Perpignan, are deduced from the Celestial Arcs given by M. DE LA CAILLE, in pag. 240, 241, *Mem. de l'Acad.* 1758, and making the proper Reductions for the Distances of the Stations of the Sector at each Place, as given in the *Mérid. vérifiée*.

SECTION

Date		Description		Amount	
1890	Jan 1	Balance		100.00	
	Feb 1	Received from A. B.		50.00	
	Mar 1	Received from C. D.		25.00	
	Apr 1	Received from E. F.		75.00	
	May 1	Received from G. H.		100.00	
	Jun 1	Received from I. J.		150.00	
	Jul 1	Received from K. L.		200.00	
	Aug 1	Received from M. N.		250.00	
	Sep 1	Received from O. P.		300.00	
	Oct 1	Received from Q. R.		350.00	
	Nov 1	Received from S. T.		400.00	
	Dec 1	Received from U. V.		450.00	
	Total			2000.00	

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S E C T I O N S E V E N T H.

An Account of the observations made during the course of the trigonometrical operation for the determination of terrestrial refraction. Plate X.

ARTICLE I. *Preamble.*

ASTRONOMICAL refraction, or that which the rays of light suffer in passing from the heavenly bodies to our earth, hath, by the investigations of different philosophers, been nearly ascertained. From the theory of dioptrics, as well as experience, it hath been proved, that the rays, in coming from a very rare into a very dense medium, are gradually bent downwards, out of their rectilinear direction, into lines more or less curved in proportion to the angular distance of the objects from the zenith, where obliquity ceasing, refraction ceases likewise; since from that point light takes the shortest route through the refracting medium to the eye of the observer. Hence it follows, that the apparent altitudes of celestial objects are greater than they otherwise would be by the quantity of this refraction, which is greatest at the horizon, amounting there to 33'.

The late Dr. BRADLEY from his experience has shewn, that in the mean state of the barometer taken at 29.6 inches, and of FAHRENHEIT's thermometer at 50°, the refraction at 45° of altitude is 57'' (according to Dr. MASKELYNE only 56'' $\frac{1}{2}$).

R

In

In other states of the atmosphere, it varies with the height of the barometer, and inversely with that of the thermometer augmented by the number 350 to the number 400.

It is therefore obvious, that terrestrial objects as well as celestial, must suffer a refraction greater or less, according as they are less or more removed from the horizon; and that supposing celestial refraction to be perfectly ascertained, the measure of the lower part of any of its curves, co-inciding with a particular object on the surface of the earth, should give the quantity by which the apparent altitude of that object would exceed its real altitude, or what would be its angle of elevation, if no such effect as refraction did exist in nature.

The instrument made use of in the triangular operation was extremely well calculated, as will be remembered from its description, for measuring with much exactness small angles of elevation or depression, and consequently was in that respect very fit for the purpose, if the multiplicity of other business we had on our hands at the time had permitted refraction to become a primary, instead of being only a secondary object. This will readily be conceived by those who have any idea of the trouble of conducting, especially at a late season of the year, an operation of the nature of that in which we were engaged. Along with the lights on the French side of the Channel, we had by day as well as by night our own inland observations to attend to; the very circulation of orders to the men posted at the different stations from twelve to fifteen or twenty miles off, in different directions around the horizon, when any part of the arrangements failed, so as to render a repetition of lights necessary, was not a matter of small detail.

But

But besides the important business of the triangles, which engrossed almost our whole attention, it is sufficiently obvious, that, in order to have been enabled to make conclusive observations, the relative heights of the stations should in strictness all have been ascertained by levelling: for purposes of this sort geometrical determinations, however good in other respects they may be, should not here be admitted, because they involve the very point in question, that is, the height, which should be obtained independently of angular measurement. Besides barometers and thermometers at both stations, two observers and two instruments of the same kind would have been necessary, for taking at the same instant the reciprocal angle of elevation and depression *.

Although, therefore, in our situation the circumstances did not admit of conclusive observations on terrestrial refraction, considered either by the mean or its extremes; nevertheless, since in a variety of cases angles both of elevation and depression were reciprocally obtained at the same stations, but at different times, it is hoped, such new light will be thrown on the

* Dr. MASKELYNE, in a letter that I lately received from him, remarks, that it would be of use to have a person to note the thermometer at the object as well as at the station of the observer, whereby (if niceties of this sort were of consequence) the refraction might be more accurately computed by the application of a new correction. Thus, calling $r = \frac{a}{10} = \frac{1}{10}$ th of the arc of distance; h = the height of the uniform atmosphere; t = the difference of the thermometers at the two stations; x = the difference of altitude of the two stations above a common level; the correction would then be $= \frac{rth}{400x}$; and the true or whole refraction would be $= r \mp \frac{rth}{400x}$, according as the thermometer stood lower or higher at the upper station.

matter as may possibly hereafter lead to further investigations of this curious, but at present vague and indetermined subject: for from these observations it will appear, that terrestrial refraction, instead of being $\frac{1}{9}$ th of the comprehended arc, according to M. BOUGUER, $\frac{1}{10}$ th according to Dr. MASKELYNE, $\frac{1}{14}$ th according to M. LAMBERT, varies from $\frac{1}{3}$ to $\frac{1}{24}$ th part of that arc; and perhaps, if it had been possible for us to have tried it on heights considerably more elevated, we should have found it almost wholly to vanish.

ART. II. *Relative heights.*

Before we proceed to give any account of the observed angles of elevation or depression, at the stations reciprocally, for trying the quantity of terrestrial refraction, it will be proper to call to remembrance, that, in the measurement of the base on Hounslow Heath, the mouth of the pipe at Hampton Poorhouse was shewn to be elevated about 60 feet above low-water spring tides at the sea, as far as could then be determined by referring it to the surface of high water at Isleworth; and that the extremity of the base near King's Arbour was found by levelling to be higher than the former end by 31 feet 3 inches.

The mouth of the pipe at the south-east end of the base of verification at High Nook near Dymchurch, in Romney Marsh, Lieut. FIDDES found by levelling to be above low-water mark at spring tides 22.1 feet.

The top of the parapet of the north turret of the Keep of Dover Castle was found by Lieut. HAY, of the Royal Engineers (by levelling from the top of the cliff at Queen Elizabeth's gun downwards, and adding to that the height of the
I
ground

ground and Castle above the said gun) to be 465.8 * feet above low water at spring tides. Having also measured a base for the purpose, he determined the height of the cliff geometrically, which agreed within less than a foot of the result by levelling. In justice to this very meritorious Officer it is incumbent on me to say, that not only on this occasion, but on every other during the progress of our operations near Dover, his assistance was most essentially useful.

The height of the ball of St. Paul's above the Thames at Paul's Wharf, and the height of Shooter's Hill Inn above the Gun Wharf in Woolwich Warren, were severally determined in 1773, at which time the experiments were carrying on for the purpose of finding a theory for the measurement of altitudes by the barometer.

The height of Severndroog Castle, lately built on Shooter's Hill, has since been deduced from that of the Inn.

Lastly, the altitudes of all the intermediate stations, as expressed in the three columns towards the right hand of the general table of results, placed at the end of the preceding section, have been established by the reciprocal angles of elevation or depression, gradually carried on from station to station, throughout the whole series of triangles, whereby the two extremities are connected together; and no greater uncertainty has been found at Hampton Poor-house than a few feet, occasioned no doubt by the uncertainty of terrestrial refraction: for it is to be remarked, that, to the westward of Greenwich, no double but only single observations were obtained; wherefore, the

* Sir THOMAS HYDE PAGE, when engineer at Dover, at my request, had been so obliging as to order his workmen (he himself being ill at the time) to determine the height of the turret of the Keep, which, by a mistake of about nine feet in the height of the cliff, they made 475 feet above low-water mark.

relative heights of these stations have been determined by taking $\frac{1}{10}$ th of the arc of distance for the effect of terrestrial refraction.

ART. III. *General Theorem.*

Let C (Plate X. fig. 11.) be the center of the earth considered as a sphere; Ss the surface; Hb two places at the same height above the surface; HO the horizontal line, or apparent level at the place H; and ho the horizontal line, or apparent level at the place b; also, let Cm bisect the angle at C.

Then, because the angles mHC and mnH are right ones, the angle mHn, or mhn, is equal to the angle mCH or mCb; that is to say, if two places H, b, are of equal heights, the one as seen from the other is depressed below the horizontal line of the place of observation, by an angle equal to half of the arc of the great circle contained between them, or half the angle at C. Hence it follows, that any distant object is higher or lower than the place of observation, according as the depression is less or greater than half the contained arc, supposing no such effect as refraction to exist in nature.

ART. IV. *Determination of the refraction between Dover Castle and Folkstone Turnpike.*

Let D (fig. 12.) be the place of the axis of the telescope on the north turret of the Keep in Dover Castle; T the ground at the station near Folkstone Turnpike; DO the horizontal line; and SL=CD.

The distance of the stations is 31554.6 feet, which, taking the obliquity of the direction into consideration, gives 61188 fathoms = 1° ; and consequently $5' 9''.4$ for the length of t

contained

contained arc of distance nearly, one-half of which, or $2' 34''.7$, is equal to the angle ODL.

At the station D, the ground at T was, by observation, elevated $8' 37''$ equal to the angle TDO, to which adding $ODL = 2' 34''.7$, we have for the angle TDL $11' 11''.7$.

Now, if the distance of the stations be taken as radius, the lines TO, TL, &c. will be nearly as the tangents of their opposite angles; therefore the angle TDL, with the distance 31554.6 give $TL = 102.7$ feet, or what the ground at the station T would have been higher than the axis of the telescope at D, if there had been no refraction.

But the axis of the telescope, when at the station T, was 5.5 feet above the ground; therefore $102.7 + 5.5 = 108.2$ feet would be the height of the axis at T above the axis at D.

Now, let T (fig. 12.) be the place of the axis of the telescope, when the instrument stood at Folkstone Turnpike; D the top of the parapet of the north turret in the Keep of Dover Castle; TO the horizontal line; and $CL = ST$.

At the station T, the parapet of the turret was by observation depressed $14' 17''.5 = OTD$, from which subtracting half the arc of distance $2' 34''.7$, there remains for the angle LTD $11' 42''.8$. This last angle, with the former known distance, give $LD = 107.5$ feet, or what the parapet was lower than the axis at T, if there had been no refraction.

But the axis of the telescope when at D was 3.2 feet above the parapet; hence $107.5 - 3.2 = 104.3$ is what the axis at D would be lower than the axis at T.

In this case it is evident, that half the sum of 108.2 and 104.3, or 106.25 feet, is the difference of the relative heights of the axis at the two stations by a mean refraction; and that this mean refraction is subtended by half the difference, or

108.2

$\frac{108.2 - 104.3}{2} = 1.95$ feet. Hence, as the distance = 31554.6 :
rad :: 1.95 : tang. $12''.8$ the mean refraction.

For suppose t (fig. 12.) to be the true place of the ground, then the elevation TDO - the refraction = TDt , or $8' 37'' - 12''.8 = 8' 24''.2$ = the angle tDO ; therefore $tDO + ODL = 10' 58''.9 = tDL$; whence $tL = 100.8$ feet, to which adding 5.5 feet, the height of the axis above the ground, we have 106.3 feet, the height of the axis at T above the axis at D as before.

Also, if d (fig. 13.) be the true place of the parapet, we shall have the depression + the refraction, or $OTD + DTd = OTd = 14' 30''.3$, and $OTd - OTL$, or $14' 30''.3 - 2' 34''.7 = 11' 55''.6$ = the angle LTd . Hence, $Ld = 109.5$ feet is what the parapet of the turret would be lower than the axis at T , from which taking 3.2 feet, the height of the axis above the parapet, there remains, as before, 106.3 feet, for the difference of the heights of the two stations.

The axis of the telescope in Dover Castle being	Feet,
above low-water spring tides	469.
To this add the height of the axis at the	
turnpike above that at Dover	106.3
	<hr/>

We then have, for the height of the axis at the	
turnpike above low water, nearly	575.3
	<hr/>

And $5' 9''.4$ the contained arc of a great circle or arc of distance being divided by $12''.8$, the mean refraction at the two stations, we have in this instance *about $\frac{1}{24}$ th part for the quantity of terrestrial refraction.*

ART. V. *Refraction on the distance between Dover Castle and Calais.*

Let D (fig. 14.) be the place of the axis of the telescope, on the north turret of the Keep in Dover Castle, as before; G the top of the great balustrade of the steeple of Notre Dame Church at Calais; DO the horizontal line; and CL = SD.

The distance of Dover from Calais by the triangles is 137450 feet, which, allowing 61169 fathoms for a degree, gives 22' 28''.2 for the length of the contained arc nearly; half of which 11' 14''.1 is equal to the angle ODL.

The height of D above low water at spring tides, as before, is 469 feet.

The height of G is 140.5

Therefore the difference is 328.5 = GL.

Then, as 137450 : rad :: 328.5 : tang. $\overset{8}{\underset{13}{\text{}}} = \text{LDG}.$

To which adding the angle ODL . . . = 11 14.1

We have 19 27.1 for the angle ODG, or what G would be depressed below the place of observation, if there was no such effect as refraction. But the depression by observation was found to be 17' 59''; wherefore the difference is 1' 28''.1, by which, dividing the length of the arc of distance 22'. 28''.2, we have in this instance 15.3, or between $\frac{1}{15}$ th and $\frac{1}{16}$ th part, for the quantity of terrestrial refraction.

ART. VI. *Refraction on the distance between Allington Knoll and Tenterden.*

Let K (fig. 15.) be the place of the axis of the telescope at Allington Knoll; T the top of the flag-staff on Tenterden Steeple;

Steeple; SC the earth's surface; KO the horizontal line at right angles to KC; and $SL = CK$.

The distance between Allington Knoll and Tenterden has by the triangles been found to be 61775.3 feet, which, allowing 61234 fathoms $= 1^\circ$ gives $10' 5''.3$ for the length of the contained arc CS nearly. Half of this arc $= 5' 2''.6$ is the angle OKL, from which subtracting the observed angle of depression of T as seen from K $= OKT = 3' 51''$, there remains the angle TKL $= 1' 11''.6$, and consequently this angle will be subtended by 21.4 feet $= LT$, or what the top of the flag-staff at Tenterden would have been higher than the axis of the telescope at the Knoll, if there had been no refraction.

But the top of the flag-staff on Tenterden steeple was 3.1 feet higher than the axis of the telescope when the instrument stood at that station; therefore, $21.4 - 3.1 = 18.3$ is what the axis at Tenterden would have been higher than the axis at the Knoll, if there had been no refraction.

Again, let T (fig. 16.) be the place of the axis of the telescope on Tenterden Steeple; K the ground at the station on Allington Knoll; TO the horizontal line; and $CL = ST$.

At the station T, the depression of the ground at K, or the angle OTK, was observed $3' 55''$, which being subtracted from $5' 2''.6 = OTL =$ half the arc of distance, there remains $1' 27''.6 =$ the angle KTL. This last angle, with the distance between the stations, 61775.3 feet, give $KL = 26.3$ feet, for what the ground at K would have been higher than the axis at Tenterden, if there had been no refraction.

But the axis of the telescope, when at the station K, was 5.5 feet above the ground; therefore, $26.3 + 5.5 = 31.8$ is what the axis at the Knoll would be higher than the axis at Tenterden.

Hence it follows, that supposing the refraction to have been the same at each of the stations when the observations were made, half the difference of these heights, or $\frac{31.8 - 18.3}{2} = 6.7$ feet, would be the difference between the relative heights of the axis at the two stations; and that the quantity of refraction would be subtended by half the sum, or $\frac{31.8 + 18.3}{2} = 25.05$ feet; therefore, to find the mean refraction, as the distance of the stations : rad. :: 25.05 feet : tang. $1' 23''\frac{1}{2}$ the mean refraction.

For supposing t (fig. 15.) to be the true place of the top of the flag-staff, we shall then have the angle OKT the depression + the angle TK t the refraction, or $3' 51'' + 1' 23''\frac{1}{2} = 5' 14''\frac{1}{2}$ = the angle OK t . Hence the angle OKT - the angle OKL = $5' 14''\frac{1}{2} - 5' 2''.6 = 0' 11''.9$ = the angle LK t . Now, this last angle, with the distance of the stations 61775.3, give Lt = 3.6 feet, or what the top of the flag-staff at Tenterden would be lower than the axis at the Knoll; and this being added to 3.1 feet (what the axis at Tenterden was lower than the top of the flag-staff), we get, as before, 6.7 feet for the height of the axis at the Knoll above the axis at Tenterden.

In like manner, supposing k (fig. 16.) to be the place of the ground at the Knoll, we have the sum of the depression and refraction, or OTK + KT k = $3' 35'' + 1' 23''\frac{1}{2} = 4' 58''\frac{1}{2}$ = OT k ; and OTL - OT k = $5' 2''.6 - 4' 58''\frac{1}{2} = 0' 4''.1$ = the angle kTL. Hence, kt = 1.2 feet is the height of the ground at the Knoll above the axis at Tenterden, which being added to 5.5 feet the height of the axis at the Knoll above the ground, we have as before 6.7 feet for the difference of the heights.

The height of the axis of the telescope at Allington Knoll above low-water mark at spring tides, as determined by the observations there, and at the station of High Nook, is Feet. 329

The axis on Tenterden Steeple has been shewn to be lower than the Knoll 6.7

Therefore, the axis on Tenterden Steeple is higher than low water 322.3

The arc of distance of the two stations = $10^{\circ} 5''.3$ being divided by $1^{\circ} 23''\frac{1}{2}$ the mean refraction, we have in this case $7''\frac{1}{4}$, or between $\frac{1}{7}$ th and $\frac{1}{8}$ th part for terrestrial refraction.

The example in Art. IV. and this last are given at large, because, if the points where the axis of the telescope was at the respective stations had been observed, in the first, one would have been a depression, and the other an elevation; but in this both would have been depressed by observation.

ART. VI. General Remarks.

The three preceding examples being sufficient to shew the mode that has been invariably adhered to in computing the effect of terrestrial refraction, we have, in the following table, collected the whole of the results together, beginning with those distances where it has been found the greatest, and ending with these where it has been found the least.

The titles at the tops of the columns respectively fully explain the nature of the table, which contains more double observations, made on a greater variety of very accurate distances, and with a better instrument for determining small angles of elevation and depression, than perhaps were ever obtained before.

fore. These results are not however offered as being free from error ; on the contrary, if the circumstances had permitted this to become a principal object in our operation, the successive repetition of the observations for many times would, no doubt, have furnished still more satisfactory conclusions. It is hoped, nevertheless, that these, such as they are, may have their use, were it only by shewing the variableness of terrestrial refraction, to induce to the making of others, which, as has been already observed, would ultimately lead to a much more minute investigation of this curious and interesting subject.

The heights of the barometer and thermometer are inserted on the days on which the observations were made, merely to shew what was nearly the state of the atmosphere at the respective times. But we have not attempted to apply any correction on that account, because it could not be done in a satisfactory manner, and consequently could not be useful, unless the circumstances had permitted reciprocal observations to have been made at corresponding times with double sets of instruments, which in our situation was impossible.

By attending to the results in the table, it will in general be seen, that terrestrial as well as celestial refraction certainly diminishes as the heights of the stations above the sea increase ; and that, at particular times at least, it is much greater than has hitherto been supposed, even to between $\frac{1}{2}$ and $\frac{1}{3}$ part of the arc of distance, instead of being only $\frac{1}{9}$ th or $\frac{1}{14}$ th part. Besides the instance of this extraordinary effect inserted in the table, between Allington Knoll and Ruckinge, where the distance of the stations is but small, and one of them little higher than the sea, we could have given another on a distance as well as on heights still more considerable, namely, Shooter's Hill and the ball of St. Paul's Church : for, supposing the
first

first to be 482, and the last 403 feet, above low water at the sea, the refraction on the morning of the 1st of September, 1787, as observed at Shooter's Hill, was $1' 47''$, which is between $\frac{1}{3}$ d and $\frac{1}{4}$ th part of the contained arc.

If the circumstances had permitted the refraction on the distance between Dover and Calais to have been repeatedly tried, at the bottom of the cliff, at the top of the cliff, and again at the top of the castle, we should probably have found the refraction at the three stations considerably different, with the same length of arc, or one only varying insensibly.

But, in order to be enabled to make conclusive observations of this sort, the operation should become a distinct one, or at most only comprehend such others as are connected with the modifications of the atmosphere. For purposes of this kind very fine levels would be requisite; and some of the highest mountains of Scotland, situated near the sea, such as *Ben Nevis* and *Cruachan Ben*, where the relative heights of the stations might be accurately ascertained by levelling, would seem to be eligible situations.

TABLE containing the RESULTS of the OBSERVATIONS for the EFFECT of TERRESTRIAL REFRACTION.

Dates of the Observations.	Places.	Bar.	Therm.	Stations.	Height of the Telescope above the Sea in Feet.	Distance of the Station in Fathoms.	Contained Arc nearly.	Mean Refraction.	Proportionable Part.
1787, Oct. 21.	Allington Knoll -	29.61	56°	Allington Knoll and Ruckinge -- --	A. 329.	2675.4	2 38	1 8	$\frac{1}{2}$ and $\frac{1}{3}$
23.	Ruckinge - -	29.82	51½		R. 37.4				
19.	Dymchurch Inn -	29.9	55½	High Nook and Lydd -- --	H. 27.6	5227.1	5 8	0 55	$\frac{1}{3}$ and $\frac{1}{6}$
21.	Allington Knoll -	29.61	56		L. 130.4				
19.	Dymchurch Inn -	29.9	55½	Allington Knoll and High Nook --	A. 329.	3864.1	3 48	0 38	$\frac{1}{6}$
21.	Allington Knoll -	29.61	56		H. 27.6				
26.	Tenterden Inn -	29.54	56½	Allington Knoll and Tenterden -- --	A. 329.	10296.	10 5	1 23½	$\frac{1}{7}$ and $\frac{1}{8}$
7.	Padleworth - -	29.6	70		T. 322.3				
1788, Aug. 18.	Frant Inn - -	29.36	58	Padleworth and Lydd -- --	P. 624.	13255.5	13 2	1 31	$\frac{1}{8}$
23.	Botley Hill - -	28.89	62½		L. 130.4				
1787, Sept. 28.	Dover Castle -	29.62	58½	Frant and Botley Hill -- --	F. 659.	15060.7	14 48	2 1	$\frac{1}{8}$
Oct. 7.	Padleworth - -	29.6	70		B. 880.				
13.	Fairlight Down -	28.81	55½	Dover Castle and Padleworth --	D. 469.	7093.5	6 57	0 42	$\frac{1}{10}$
26.	Tenterden Inn -	29.54	56½		P. 642.				
26.	Tenterden Inn -	29.54	56½	Fairlight Down and Tenterden --	F. 599.	11939.1	11 46	1 12	$\frac{1}{10}$
1788, Aug. 11.	Goudhurst Churchyard	29.74	58½		T. 322.3				
18.	Frant Inn - -	29.36	58	Tenterden and Lydd -- --	T. 322.3	11027.8	10 50	0 55½	$\frac{1}{11}$ and $\frac{1}{12}$
1787, Oct. 13.	Fairlight Down -	28.81	55½		L. 130.4				
1788, Aug. 11.	Goudhurst Churchyard	29.74	58½	Goudhurst and Frant -- --	G. 497.	7398.3	7 15	0 38½	$\frac{1}{11}$ and $\frac{1}{12}$
1787, Oct. 26.	Tenterden Inn -	29.54	56½		F. 659.				
21.	Allington Knoll -	29.61	56	Fairlight Down and Lydd -- --	F. 599.	11948.3	11 43	0 55	$\frac{1}{13}$
Sept. 28.	Dover Castle - -	29.62	58½		L. 130.4				
1788, Sept. 2.	Folkstone Turnpike	29.55	64	Goudhurst and Tenterden -- --	G. 497.	9062.4	8 53	0 35	$\frac{1}{15}$
					T. 322.3				
				Allington Knoll and Lydd -- --	A. 329.	7975.	7 51	0 21	$\frac{1}{12}$
					L. 130.4				
				Dover Castle and Folkstone Turnpike	D. 469.	5259.1	5 9.4	0 12.8	$\frac{1}{14}$
					F. 575.3				

RESULTS of SINGLE OBSERVATIONS depending on the Heights of Dover Castle and Folkstone Turnpike, combined with those on the Coast of France.

1788, Sept. 2.	-- -- --	29.55	64	Folkstone Turnpike and N. D. Church at Calais	F. 575.3	26697.	26 4	3 54.8	$\frac{1}{6}$ and $\frac{1}{7}$
1787, Sept. 28.	-- -- --	29.62	58½		C. 140.5				
1788, Sept. 2.	-- -- --	29.55	64	Dover Castle and N. D. Church at Calais	D. 469.	22908.	22 28.2	0 15.4	$\frac{1}{13}$ and $\frac{1}{16}$
					C. 140.5				
				Folkstone Turnpike and Station at Montlambert	F. 575.3	28967.	28 29	1 45.	$\frac{1}{17}$
					M. 639.5				

SECTION EIGHTH.

Secondary triangles, subdivided into two sets, for the improvement of the maps of the country, and the plan of the City of London and its Environs. Plate XI.

IN the series of great triangles, whereby the distance between the meridians of the Royal Observatories of Greenwich and Paris has been determined, the same excellent instrument having been placed at every station on our side of the Channel, and all the angles observed with the utmost care, it hath consequently followed, that the base on Hounslow Heath, and that in Romney Marsh, reciprocally measure each other within a few inches of the truth, which is an instance of such exactness as probably never occurred in any former operation of this sort. The extreme smallness of the error on the sum of the three angles of each triangle sufficiently proves that the general result would not have differed greatly, if only two of the angles had actually been observed. But in an operation of so much importance, this could not have been depended upon; nothing was to be left doubtful; and therefore, in the execution of the various parts, the most minute attention was paid to every circumstance whereby the accuracy might be affected, and particularly to the placing of the lights and instrument reciprocally over the same point marking the station, that no possible error might arise from parallax or excentricity.

From

From this mode of conducting the operation, it will readily be seen, that, if time had permitted, the situation of a multitude of other points in the country might have been very accurately determined, besides those actually marking the points of the triangles, whereby the ordinary maps would have been greatly improved by such as chose at any time hereafter to make use of these as so many given distances. But the circumstances not having permitted us to multiply those points to the extent that might have been wished, and that would have been easily practicable, if the operation had commenced at an earlier season of the year; we have therefore been obliged to limit the number to a few of the most conspicuous and best defined objects.

These secondary triangles are subdivided into two sets. The first set consists of thirty-five, whereby the relative distances of so many points have been determined from certain stations of the principal series, beginning with those objects that have been intersected from the most westerly stations, and so on, proceeding gradually with the others towards the east. Two angles only of each of those triangles being observed, the third is that at the intersected object, or the supplement to 180° . Although the distances thus obtained cannot be quite so accurate as the sides of the principal series; yet there is no reason to apprehend, that they will be found to differ widely from the truth, when they come to be proved in the course of any subsequent operation, by which alone they can be put to the test.

Computation of the first set of secondary triangles.			
N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.
1.	<div>King's Arbour .</div> <div>St. Ann's Hill .</div> <div>Stanwell Church .</div>	<div>8° 52' 57"</div> <div>4 4 44</div> <div>167 2 19</div>	<div>} from Stanwell {</div> <div>10927</div> <div>23720</div>
2.	<div>King's Arbour .</div> <div>Hanger-hill Tower .</div> <div>Harrow on the Hill .</div>	<div>28 35 34</div> <div>89 23 52</div> <div>62 0 34</div>	<div>} from Harrow on the Hill {</div> <div>42944</div> <div>20553.3</div>
3.	<div>King's Arbour .</div> <div>Hanger-hill Tower .</div> <div>Banstead Church .</div>	<div>70 1 47</div> <div>82 19 25.1</div> <div>27 38 47.9</div>	<div>} from Banstead Church {</div> <div>80994</div> <div>76807.5</div>
4.	<div>Hampton Poor-house .</div> <div>King's Arbour .</div> <div>Kew Pagoda .</div>	<div>88 58 23</div> <div>40 14 25</div> <div>50 47 12</div>	<div>} from Kew Pagoda . {</div> <div>22849</div> <div>35364.5</div>
5. †	<div>Harrow on the Hill .</div> <div>St. Paul's Church .</div> <div>Spring Grove House, Sir Jo. BANKS * .</div>	<div>69 43 8</div> <div>35 58 9</div> <div>74 18 43</div>	<div>} from Spring Grove House {</div> <div>35851</div> <div>57253.9</div>
6.	<div>Hanger hill Tower .</div> <div>Spring Grove House * .</div> <div>Richmond Royal Observatory</div>	<div>19 33 4.3</div> <div>82 46 15.9</div> <div>77 40 39.8</div>	<div>{ from Richmond Royal Observatory {</div> <div>20164.4</div> <div>6802.1</div> <div>Hanger Hill from Spring Grove . . . 19857.8</div>

† The Royal Observatory in Richmond lower Park could not be seen from any of the stations of the great series of triangles, except Hanger-hill Tower, from whence the bearing of it was taken. In order to intersect this bearing, the assistance of certain operations made with the astronomical quadrant in 1783 at Spring Grove House has been called in, by the help of which the situations of the Observatory and of Spring Grove House have been determined. In like manner, the bearings of Battersea and Stretham, taken from Hundred Acres, have been intersected with the quadrant from St. Paul's and Fulham. The stations where the quadrant was used are distinguished with asterisks.

N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.
7.	<div><div>Hundred Acres . .</div><div>St. Paul's * . .</div><div>Battersea Church . .</div></div>	<div><div>° ' "</div><div>14 13 27</div><div>34 3 49.2</div><div>131 42 43.8</div></div>	<div><div>} from Battersea Church {</div><div>50664.5</div><div>22226</div></div>
8.	<div><div>Hundred Acres . .</div><div>Fulham Church * . .</div><div>Stretham Church . .</div></div>	<div><div>27 51 55.6</div><div>46 12 54.4</div><div>105 55 10</div></div>	<div><div>} from Stretham Church {</div><div>35957.3</div><div>23279.3</div></div>
9.	<div><div>Hundred Acres . .</div><div>Severndroog Castle . .</div><div>Clapham Common, Mr. CA- VENDISH . .</div></div>	<div><div>36 59 35.8</div><div>33 28 20.5</div><div>109 32 3.7</div></div>	<div><div>} from Clapham Common {</div><div>43351.7</div><div>47296.4</div></div>
10.	<div><div>Norwood . .</div><div>Severndroog Castle . .</div><div>Argyll Street Ob. Maj. Gen. Roy</div></div>	<div><div>76 19 14.5</div><div>52 41 37</div><div>50 59 8.5</div></div>	<div><div>} from Argyll Street Ob- servatory {</div><div>40083.2</div><div>39963</div></div>
11.	<div><div>Norwood . .</div><div>Severndroog Castle . .</div><div>St. Paul's Church . .</div></div>	<div><div>62 30 23.5</div><div>57 8 8.5</div><div>60 21 28</div></div>	<div><div>} from St. Paul's Church {</div><div>37840.9</div><div>39963</div></div>
N. B. By combining the results of these two last triangles a third is formed, which gives for the distance of Argyll Street from St. Paul's			
			9632
12.	<div><div>Norwood . .</div><div>Severndroog Castle . .</div><div>Bromley College . .</div></div>	<div><div>36 36 32</div><div>32 52 48</div><div>110 30 40</div></div>	<div><div>} from Bromley College {</div><div>22695.4</div><div>24930.6</div></div>
13.	<div><div>Norwood . .</div><div>Severndroog Castle . .</div><div>Chislehurst Church . .</div></div>	<div><div>31 53 3</div><div>67 48 12.5</div><div>80 18 44.5</div></div>	<div><div>} from Chislehurst Church {</div><div>35777.9</div><div>20981.1</div></div>
14.	<div><div>Greenwich Royal Observatory</div><div>Severndroog Castle . .</div><div>West Pediment of Wanstead Ho</div></div>	<div><div>92 38 13.5</div><div>64 46 33.5</div><div>22 35 13</div></div>	<div><div>} from Wanstead House {</div><div>34413.6</div><div>37999.7</div></div>
15.	<div><div>Greenwich Royal Observatory</div><div>Severndroog Castle . .</div><div>Loampit Hill . .</div></div>	<div><div>131 45 43</div><div>14 7 0</div><div>34 7 17</div></div>	<div><div>} from Loampit Hill {</div><div>6352.6</div><div>19428.4</div></div>
16.	<div><div>Greenwich Royal Observatory</div><div>Severndroog Castle . .</div><div>Beckenham Church . .</div></div>	<div><div>85 49 9</div><div>63 29 48</div><div>30 41 3</div></div>	<div><div>} from Beckenham Church {</div><div>25622</div><div>28555</div></div>

N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.
17.	Greenwich Royal Observatory Severndroog Castle Eltham Church	$\begin{array}{r} 22^{\circ} 41' 33'' \\ 87^{\circ} 18' 31.5'' \\ 69^{\circ} 59' 55.5'' \end{array}$	} from Eltham Church { $\begin{array}{r} 15531 \\ 5998.3 \end{array}$
18.	Severndroog Castle Botley Hill Knockholt Beeches	$\begin{array}{r} 21^{\circ} 56' 44'' \\ 54^{\circ} 48' 27'' \\ 103^{\circ} 14' 49'' \end{array}$	} from Knockholt Beeches { $\begin{array}{r} 58933 \\ 26951 \end{array}$
19.	Severndroog Castle Botley Hill Leeth Hill Tower	$\begin{array}{r} 31^{\circ} 40' 29.4'' \\ 124^{\circ} 53' 14'' \\ 23^{\circ} 26' 16.6'' \end{array}$	} from Leeth Hill Tower { $\begin{array}{r} 144761 \\ 92668 \end{array}$
20.	Botley Hill Frant Church Firedean Tower	$\begin{array}{r} 39^{\circ} 17' 6.5'' \\ 26^{\circ} 58' 39'' \\ 113^{\circ} 44' 4.5'' \end{array}$	} from Firedean Tower { $\begin{array}{r} 44780.4 \\ 62507 \end{array}$
21.	Botley Hill Frant Church Crowborough Beacon	$\begin{array}{r} 19^{\circ} 51' 19.5'' \\ 77^{\circ} 32' 33'' \\ 82^{\circ} 36' 7.5'' \end{array}$	} from Crowborough Beacon { $\begin{array}{r} 88977 \\ 30949.7 \end{array}$
22.	Botley Hill Wrotham Hill Sevenoaks Windmill	$\begin{array}{r} 24^{\circ} 22' 7'' \\ 28^{\circ} 57' 42'' \\ 126^{\circ} 40' 11'' \end{array}$	} from Sevenoaks Windmill { $\begin{array}{r} 44032.4 \\ 37519.8 \end{array}$
23.	Frant Church Goudhurst Church Wadhurst Church	$\begin{array}{r} 46^{\circ} 5' 9'' \\ 26^{\circ} 21' 46.5'' \\ 107^{\circ} 33' 4.5'' \end{array}$	} from Wadhurst Church { $\begin{array}{r} 20674 \\ 33538.7 \end{array}$
24.	Goudhurst Church Fairlight Down Brightling Windmill	$\begin{array}{r} 42^{\circ} 6' 25'' \\ 38^{\circ} 5' 33'' \\ 99^{\circ} 48' 2'' \end{array}$	} from Brightling Windmill { $\begin{array}{r} 58616.3 \\ 63707.4 \end{array}$
25.	Fairlight Down Lydd Church Rye Church	$\begin{array}{r} 22^{\circ} 40' 17'' \\ 21^{\circ} 23' 25'' \\ 135^{\circ} 56' 18'' \end{array}$	} from Rye Church { $\begin{array}{r} 37598 \\ 39734 \end{array}$
26.	Fairlight Down Dover Castle, north turret Dengeneffs Light-house	$\begin{array}{r} 19^{\circ} 34' 30'' \\ 13^{\circ} 54' 24.6'' \\ 146^{\circ} 31' 5.4'' \end{array}$	} from Dengeneffs Light-house { $\begin{array}{r} 81082.7 \\ 113030 \end{array}$
27.	Fairlight Down Goudhurst Church Ore Church	$\begin{array}{r} 4^{\circ} 12' 42'' \\ 60^{\circ} 29' 28'' \\ 115^{\circ} 17' 50'' \end{array}$	} from Ore Church { $\begin{array}{r} 7605.3 \\ 90123.2 \end{array}$

N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.
28.	Fairlight Down . Lydd Church . <i>Fairlight Church</i> .	⁰ 32' 23" 1 50 43 154 36 54	} from Fairlight Church { 5384 } 66787.7
29.	Tenterden Church . Allington Knoll . <i>Ashford Church</i> .	30 42 37 46 45 7 102 32 16	} from Ashford Church { 46096 } 32319.2
30.	Lydd Church . High Nook near Dymchurch <i>Ruckinge Church</i> .	43 34 50.5 87 40 22 48 44 47.5	} from Ruckinge Church { 41682.2 } 28758.4
31.	High Nook . Ruckinge . <i>New Romney Church</i> .	83 44 33.5 32 17 35.5 63 57 51	} from New Romney Ch. { 16965.4 } 31566.3
32.	High Nook . Allington Knoll . <i>Lymne Castle</i> .	42 44 44.5 70 21 48 66 53 27.5	} from Lymne Castle { 23741.6 } 17109.6
33.	Lydd Church . Folkstone Turnpike . <i>Folkstone Church</i> .	2 10 29.2 27 26 22 150 23 8.8	} from Folkstone Church { 78946.9 } 6501.3
34.	Folkstone Turnpike . Padlesworth . <i>Beachborough Summer House</i>	24 35 59 123 46 35.2 31 37 25.8	} from Beachborough Summer House { 23325.2 } 11681.6
35.	Padlesworth . Dover Castle . <i>Waldershare Monument</i> .	32 36 0 62 24 5 84 59 55	} from Waldershare Mo- nument { 37862.5 } 23081.4

Second set of secondary triangles.

In the Paper of 1787, sufficient reasons have been given for avoiding St. Paul's as a station in the series of great triangles. Indeed, if no other objection had existed, the smoke of the capital alone would have been found extremely inconvenient. This was experienced at Shooter's Hill, where we

were

were detained a whole week, before the white lights, notwithstanding their extraordinary brilliancy, could be seen at Hanger-hill Tower, or even at Argyll Street, the north-east wind, which then prevailed, having brought the impenetrable mafs of fmoke between the ftation of the instrument and the points to be obferved; and at laft we were obliged to watch all night, till towards the morning the fires of London being extinguifhed, the white lights could then be interfected.

It is not therefore furprifing, that from the ftations of Norwood, Greenwich, and Shooter's Hill, we fhould only be able to fix, in a fatisfactory manner, two points in London, namely, St. Paul's and Argyll Street. Bearings, it is true, of others were taken; but that thefe might be interfected by angles not too acute, it became neceffary to make ufe of obfervations that had been formerly obtained at Argyll Street with my own instrument in its vertical pofition, and at St. Paul's with the astronomical quadrant. Moreover, by way of finishing the operation, and furnifhing fuch part of the inhabitants of the metropolis as may be curious in matters of this fort with a fet of diftances that cannot fail to be ufeful to them, two new ftations were chofen for the great instrument to the northward of London, one on *Hornfey Hill*, and the other on *Primrofe Hill*. Thus, from the combined operations at thefe feveral places, we have been able to determine the fituation of thirty-seven conspicuous points, confifting chiefly of the moft remarkable fteeples in and near the capital.

By referring to Plate XI. which is in fact the fkeleton, but on a very fmall fcale, for an improved plan of London and its environs, the relative fituation of thefe points with regard to St. Paul's, and the four neareft ftations of the great feries, will be feen. Some of the principal of thefe fecondary triangles

angles have been represented by dotted lines in the plan. To have expressed more of them in that way would only have occasioned confusion. Here it is to be remarked, that the distance of Argyll Street from St. Paul's, 9632 feet, resulting from the 10th and 11th secondary triangles of the first set, becomes a base in the quadrilateral formed by *St. Paul's*, *Argyll Street*, *Hornsey Hill*, and *Primrose Hill*. Hence, by the observed angles at these two last stations, and the assumed length of one of the unknown sides, all the angles of the quadrilateral are computed, by which means, and the true length of one side given, the true lengths of all the others are readily obtained.

Computation of the second set of secondary triangles.

Situations determined by the great instrument from Hornsey Hill and Primrose Hill.	N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.	
	1.	Hornsey Hill Primrose Hill <i>St. Paul's</i>	46° 42' 41" 83 21 27.5 49 55 51.5	} from <i>St. Paul's</i> Church Hornsey Hill from Primrose Hill	23297.1 17072.8
					17949.05
	2.	Hornsey Hill Primrose Hill. <i>Argyll Street Observatory</i>	23 8 34 112 49 57 44 1 29	} <i>Argyll Street</i> Observatory	23803.4 10150.7
	3.	Hornsey Hill Primrose Hill <i>Hampstead Church</i>	23 33 59 78 23 43 78 2 18	} from <i>Hampstead</i> Church	17972 7335.5
	4.	Hornsey Hill Primrose Hill <i>Mr. DUVELUZ's Cupola</i> , Hornsey Lane, Highgate	29 11 3.5 16 44 50 134 4 6.5	} from <i>Mr. DUVELUZ's</i> Cupola	12181.2 7198.2
	5.	Hornsey Hill Primrose Hill <i>Islington Church</i>	47 30 42 51 42 39 80 46 39	} from <i>Islington</i> Church	14272.5 13409

With the great instrument from Hornsey Hill and Primrose Hill.	N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.	
	6.	Hornsey Hill . Primrose Hill . <i>Highbury House, Mr. AUBERT</i>	55° 28' 32" 29 28 52 95 2 36	} from Highbury House	{ 8867.7 14845.4
	7.	Hornsey Hill . Primrose Hill . <i>St. Luke's Church, Old Street</i>	50 52 33 68 59 37 60 7 50	} from St. Luke's Church	{ 19323 16057.5
	8.	Hornsey Hill . Primrose Hill . <i>St. Leonard's Ch. Shoreditch</i>	62 9 30 63 36 33.5 54 13 56.5	} from St. Leonard's Church	{ 19816 19560.5
	9.	Hornsey Hill . Primrose Hill . <i>Christ Church, Spitalfields</i>	61 19 7.5 70 33 39 48 7 13.5	} from Christ Church, Spit.	{ 22733 21149.3
	10.	Hornsey Hill . Primrose Hill . <i>Bow Church, Cheapside</i>	49 20 38.5 81 21 4.5 49 18 17	} from Bow Ch.	{ 23404.4 17959.6
	11.	Hornsey Hill . Primrose Hill . <i>St. Bride's Ch. Fleet Street</i>	42 33 43 86 44 24 50 41 53	} from St. Bride's Church	{ 23158 15689.1
	12.	Hornsey Hill . Primrose Hill . <i>St. George's Ch. Bloomsbury</i>	31 16 11 94 11 25 54 32 24	} from St. George's Church	{ 21977 11438
	13.	Hornsey Hill . Primrose Hill . <i>St. Giles's Church</i>	29 32 8.5 100 13 30.5 50 14 21	} from St. Giles's Church	{ 22978.4 11510.4
	14.	Hornsey Hill . Primrose Hill . <i>St. Ann's, Scho</i>	28 2 38 106 40 20 45 17 2	} from St. Ann's Church	{ 24197.1 11875.4
One angle taken with the great instrument, and the other with that in Argyll Street Obs.	15.	Hornsey Hill . Argyll Street Observatory <i>Highgate Chapel</i>	59 52 55.5 22 37 47.6 97 29 16.9	} from Highgate Chapel	{ 9237.8 20766.9
	16.	Primrose Hill . Argyll Street Observatory <i>St. Clement's Church</i>	20 43 50 123 0 9.4 36 16 0.6	} from St. Clement's Church	{ 14390.2 6073.8

	N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.
One angle taken with the great instrument, and the other with that in Argyll Street Observat.	17.	Primrose Hill Argyll Street <i>St. Mary's Ch. in the Strand</i>	$\begin{matrix} 17 & 52 & 31 \\ 127 & 21 & 15 \\ 34 & 46 & 14 \end{matrix}$	$\left. \begin{matrix} \text{from St. Mary's} \\ \text{Church} \end{matrix} \right\} \begin{matrix} 14148.5 \\ 5463.4 \end{matrix}$
	18.	Primrose Hill Argyll Street Observat. <i>St. Martin's Ch. in the Fields</i>	$\begin{matrix} 7 & 32 & 8.5 \\ 152 & 0 & 27 \\ 20 & 27 & 24.5 \end{matrix}$	$\left. \begin{matrix} \text{from St. Martin's} \\ \text{Church} \end{matrix} \right\} \begin{matrix} 13631.6 \\ 3808.8 \end{matrix}$
	19.	Primrose Hill Argyll Street Observat. <i>Pantheon</i>	$\begin{matrix} 3 & 12 & 59.5 \\ 102 & 32 & 39.4 \\ 74 & 14 & 21.1 \end{matrix}$	$\left. \begin{matrix} \text{from the Pan-} \\ \text{theon} \end{matrix} \right\} \begin{matrix} 10295.5 \\ 591.8 \end{matrix}$
A small HADLEY's sextant used in Argyll Street.	20.	Primrose Hill Argyll Street Observat. <i>St. George's Ch. Hanover Sq.</i>	$\begin{matrix} 5 & 35 & 34 \\ 120 & 13 & 56 \\ 54 & 10 & 30 \end{matrix}$	$\left. \begin{matrix} \text{from St. George's} \\ \text{Church} \end{matrix} \right\} \begin{matrix} 10816.5 \\ 1220.1 \end{matrix}$
	21.	Primrose Hill Argyll Street Observat. <i>South Audley Chapel</i>	$\begin{matrix} 16 & 7 & 10 \\ 103 & 34 & 59 \\ 60 & 17 & 51 \end{matrix}$	$\left. \begin{matrix} \text{from South Aud-} \\ \text{ley Chapel} \end{matrix} \right\} \begin{matrix} 11359.3 \\ 3244.5 \end{matrix}$
A small HADLEY's sextant used at St. Paul's.	22.	Hornsey Hill St. Paul's Church <i>Newington Church</i>	$\begin{matrix} 38 & 14 & 6 \\ 16 & 35 & 7 \\ 125 & 10 & 47 \end{matrix}$	$\left. \begin{matrix} \text{from Newington} \\ \text{Church} \end{matrix} \right\} \begin{matrix} 8136.4 \\ 17641 \end{matrix}$
	23.	Hornsey Hill St. Paul's Church <i>St. Matthew's Church, Beth- nal Green</i>	$\begin{matrix} 20 & 29 & 59 \\ 66 & 7 & 5 \\ 92 & 22 & 56 \end{matrix}$	$\left. \begin{matrix} \text{from St. Mat-} \\ \text{thew's Church} \end{matrix} \right\} \begin{matrix} 21321 \\ 8165.8 \end{matrix}$
The astronomical quadrant used at St. Paul's in 1783.	24.	Hornsey Hill St. Paul's Church <i>St. George's, Ratcliff</i>	$\begin{matrix} 18 & 22 & 9 \\ 105 & 32 & 24 \\ 56 & 5 & 27 \end{matrix}$	$\left. \begin{matrix} \text{from St. George's} \\ \text{Church, Ratcliff} \end{matrix} \right\} \begin{matrix} 27045.2 \\ 8846.4 \end{matrix}$
	25.	Primrose Hill St. Paul's Church <i>St. James's Church</i>	$\begin{matrix} 30 & 44 & 17 \\ 45 & 39 & 31 \\ 103 & 36 & 12 \end{matrix}$	$\left. \begin{matrix} \text{from St. James's} \\ \text{Church} \end{matrix} \right\} \begin{matrix} 12562.7 \\ 8978 \end{matrix}$
	26.	Greenwich Royal Observat. St. Paul's Church <i>Limehouse Church</i>	$\begin{matrix} 31 & 5 & 38 \\ 27 & 52 & 40 \\ 121 & 1 & 42 \end{matrix}$	$\left. \begin{matrix} \text{from Limehouse} \\ \text{Church} \end{matrix} \right\} \begin{matrix} 13999.3 \\ 15462 \end{matrix}$

	N ^o	Triangles.	Angles.	Distance of the stations from the intersected object in feet.	
The astronomical quadrant used at St. Paul's in 1783.	27.	Argyll Street Observatory	61° 47' 27"	} from St. Peter's Church, Westm. {	6279.5 8661.8
		St. Paul's Church .	39 42 24.5		
		S.W. pinnacle of the S. tower of St. Peter's Ch. Westm.	78 30 8.5		
Great inf. and Argyll Street instrument.	28.	Norwood . . .	18 5 5	} from the Monument {	36409.7 12557.5
		Argyll Street Observatory <i>The Monument</i> .	64 9 55.7 97 44 59.3		
Both with the astronomical quadrant.	29.	Jew's Harp station .	52 52 53	} from St. Paul's Church { from Jew's Harp to Black Lane, the base of 1783 .	13522.0 10790.3 7744.3
		Black Lane station .	92 12 30		
		<i>St. Paul's Church</i> .	34 54 37		
One angle with the Argyll Str. instrument and the other with the astronomical quadrant.	30.	Jew's Harp station .	89 56 55.9	} from Argyll Str. {	5656.8 9586.2
		Black Lane station .	36 9 50		
		<i>Argyll Street Observatory</i>	53 53 14.1		
	31.	Argyll Street Observatory	95 30 56.5	} from the British Museum {	3488.3 6511
		Jew's Harp station .	30 5 26		
		<i>Wind Vane of the British Museum</i> .	54 23 37.5		
32.	Argyll Street Observatory	74 26 16.6	} from Charlotte Street Chapel {	1848.1 5459.4	
	Jew's Harp station .	19 1 55.9			
	<i>Charlotte Street Chapel</i> .	86 31 47.5			
Both angles observed with the astronomical quadrant in 1783.	33.	Jew's Harp station .	85 27 45	} from Portland Chapel {	4097.7 8474.2
		Black Lane station .	28 53 30		
		<i>Portland Chapel</i> .	65 38 45		
	34.	Jew's Harp station .	60 43 55	} from Fitzroy Chapel {	4015.5 6759.6
		Black Lane station .	31 12 45		
		<i>Fitzroy Chapel</i> .	88 3 20		
	35.	Jew's Harp station .	63 25 50	} from the Tabernacle {	4780.8 7048.4
		Black Lane station .	37 14 40		
		<i>Tabernacle</i> .	79 19 30		
	36.	Jew's Harp station .	19 45 45	} from the Small-Pox Hospital {	6670.5 2690.4
		Black Lane station .	56 57 45		
		<i>Small-Pox Hospital</i> .	103 16 30		

Both angles observed with the alt. quadrant in 1783.	N ^o	Triangles.	Angles.	Distances of the stations from the intersected object in feet.	
			^o ['] ["]		
{	37.	Jew's Harp station .	4 41 45	} from St. Pancras Church {	5728.1
		Black Lane station .	12 58 25		2088.8
		St. Pancras Church .	162 19 50		

That these secondary triangles may be more generally useful to the inhabitants of London and its environs, the angles, which the 53 points comprehended in Plate XI. respectively form with each other at the center of the dome of St. Paul's, are collected in the annexed table, together with their several distances from that central point. The objects are arranged into two classes, according as they are situated to the eastward or westward of the meridian of St. Paul's. Those of the first class commence at the north meridian, and proceed by the east to 180°. These of the second commence at the south meridian, and proceed, in like manner, by the west to 180°. From this table the total angle between any two objects being had by simple subtraction, and the distances from St. Paul's given, the distances of the objects from each other are readily obtained. Whoever, therefore, should be desirous of knowing accurately his own situation in this great metropolis may easily satisfy himself, by taking two angles from the top of his house, with a good HADLEY's sextant or theodelet, between any known objects near to him, and the best disposed for the purpose. By the help of these *data*, and a very simple trigonometrical computation, he will obtain what he wants; and he may even satisfy another curiosity which will probably occur, namely, that of putting to the test our original operation, by trying how nearly different triangles bring out the same result. It will readily be conceived that, for trials of this sort, the

points.

points whose situations have been determined by the great instrument should be chosen preferably to the others; and next to these, the objects that have been fixed by one angle, taken with the Argyll-Street instrument, as more to be relied upon, than those observed with the astronomical quadrant or sextant. Thus an excellent foundation is laid for the improvement of the plan of London and its environs, which may by these triangles be rendered more accurate than would have been possible by any other mode.

Table shewing the bearings and distances of objects situated in and near London, from the center of the Dome of St. Paul's.

	Objects.	Bearings from the north me- rid. eastward.	Distances in feet.
Eastward of the meridian of St. Paul's.	Newington Church	9 59 39.3	17641
	St. Luke's Church, Old Street	12 57 53.7	4262.7
	St. Leonard's Church, Shoreditch	44 57 39.8	6746.4
	The west pediment of Wanstead House	55 53 46.4	36308.4
	St. Matthew's Church, Bethnal Green	59 31 37.3	8165.8
	Christ's Church, Spitalfields	70 38 37.3	5878.4
	Bow Church, Cheapside	87 48 4.1	1078.1
	Limehouse Church	92 51 6	15462
	St. George's Church, Ratcliff Highway	98 56 56.3	8846.4
	The Monument	115 15 45.7	3114.2
	Severndroog Castle, Shooter's Hill	115 28 50.4	39963
	Transit-room of Greenwich Royal Observ.	120 43 46	25655.5
	Eltham Church	123 46 4.2	41091
	Loampit Hill	134 40 48.7	23450.1
	Station at Norwood	175 47 18.4	37841
	South Meridian of St. Paul's	180 0 0	— —

	Objects.	Bearings from the south me- rid. westward.	Distances in feet.
Westward of the meridian of St. Paul's.	Stretham Church	13 57 7.8	31793.5
	Clapham Common, Mr. CAVENDISH	26 29 56.1	24563.5
	Battersea Church	52 22 27.6	22226
	St. Peter's Ch. Westm. SW pinnacle of the S. T.	52 32 13.2	8661.8
	Fulham Church	57 39 44.6	30746.3
	Kew Pagoda	71 2 36	47577.7
	Richmond, Royal Observatory	71 42 0.1	51941.1
	St. Martin's Church, in the Strand	74 28 59.2	6748.6
	Spring Grove House, Sir Jos. BANXS, Bt.	76 9 49.3	57253.9
	St. James's Church	77 49 9.8	8978
	South Audley Chapel	81 49 42.7	12211.1
	St. Mary's, New Church in the Strand	81 57 27.7	4291.6
	St. Clement's Church	85 57 36.7	3592.4
	St. Ann's Church, Soho	86 9 58.7	7753.9
	St. George's Church, Hanover Square	86 23 12.9	10304.5
	St. Bride's Church, Fleet Street	90 12 42.8	1771.7
	Argyll Street Observatory, Maj. Gen. Roy	92 14 37.7	9632
	The Pantheon	93 19 17.2	9066.8
	Hanger-hill Tower	94 24 59.6	45845.2
	St. Giles's Church	94 36 28.3	6917.3
	Portland Chapel	100 34 38.7	10301.4
	Charlotte Street Chapel	101 30 23.9	8500.4
	St. George's Church, Bloomsbury Square	103 15 20.6	6221.1
	Wind Vane of the British Museum	105 45 45.7	6701.5
	The Tabernacle, Tottenham Court Road	107 20 47	8876.2
	Fitzroy Chapel	109 41 10.2	9559.9
	Harrow on the Hill Church	112 7 58.3	58764.2
	Jew's Harp station of 1783	112 58 30.7	13522
	Primrose Hill station of 1788	123 28 40.8	17072.8
	Hampstead Church	128 56 11.8	24148.7
	St. Pancras Church	136 43 25.8	10600.7
	The Small-Pox Hospital	137 38 37.8	8732.3
	Black Lane station of 1783	147 53 7.7	10790.3
	Highgate Chapel	150 59 18.1	24062.4
	Mr. DUVELUZ's Cupola, Hornsey Lane, High.	155 27 12.8	22646
	Hornsey Hill station of 1788	173 24 32.3	23297.1
	Islington Church	174 40 21.4	9028.2
	Highbury House, Mr. AUBERT	178 43 11.6	14595.7
	Ditto, the Transit-room of his Observatory	179 1 56.6	14561.4
	North meridian of St. Paul's	180 0 0	- -

Computed latitudes and longitudes of some of the places in the above table.

Places.	Latitudes.	Longitude from Greenwich	
		in degr. &c.	in time.
	° ' "	° ' "	h. m. sec.
St. Paul's	51 30 49.43	0 5 46.8	0 0 23.12
Highbury House, Transit-room	51 33 12.8	0 5 50.5	0 0 23.37
St. James's Church	51 30 30.7	0 8 5	0 0 32.33
Argyll Street Observatory	51 30 53.05	0 8 18.36	0 0 33.224
Clapham Common, Transit-room	51 27 12.7	0 8 39.2	0 0 34.613
Richmond Royal Observatory	51 28 7.9	0 18 42.3	0 1 14.82

CONCLUSION.

IN the course of this Paper, an account has been given of the commencement, progress, and completion of an operation, the first of its kind in this country, undertaken by the command, and executed under the auspices, of a most gracious and beneficent Sovereign, the Patron of the Sciences.

From a liberal supply of much better instruments than ever were used for purposes of this sort on any former occasion, and every other assistance that could contribute towards success, the operation has undoubtedly derived some peculiar advantages: for, besides a more accurate mode of measuring the bases than has heretofore been practised, the angles of the triangles have been observed so truly, that the relative geodetical situations of the stations, as determined by plane trigonometrical computation, may be said to be free from sensible error.

The instrument too, by means of its transit telescope, being admirably calculated for determining, with great precision, the true direction of the meridians, their convergence to each other, and consequently the differences of longitude, have thereby

thereby been obtained by angular measurement alone, without any regard to difference of time, more or less erroneous even with the very best time-keepers, and not perhaps to be depended upon to nearer than half a second, after taking a mean of a number of comparisons. This mode, by angular measurement, was suggested in the Paper of 1787; and we presume to think, that the result of the operation has fully verified the goodness of the method by the consistency of the pole-star observations among themselves. It may be said to be a new mode of surveying, by the help of the pole-star as a fixed point, for preserving the accuracy of the operation, successively carried on from meridian to meridian; and the same mode should be adhered to in future.

Another circumstance must likewise be noticed, as having been proposed at the same time, namely, the use of white lights for the distant stations: for without the help of these, observed with such an instrument as ours, it would have been utterly impossible to have determined accurately the distances of Montlambert and Blancnez, the first nearly forty-seven, and the last nearly forty-eight miles from Fairlight Down.

Without farther recapitulation, the Writer of this Account cannot help considering it as being incumbent on him to recommend, that the trigonometrical operation, so successfully begun, should certainly be continued, and gradually extended over the whole island. Compared with the greatness of the object, the annual expence to the publick would be a mere trifle not worthy of being mentioned. In reality, a chief part of the expence, namely, that of fine instruments, has already been incurred; and it would be a pity indeed to suffer them to be laid up and remain useless. The honour of the nation is concerned in having at least as good a map of this as there is of
any

any other country. But, by proceeding with the work in the same manner as it has been begun, with more perfect instruments than have heretofore been used, and some of these applied in a new way, a map of the British islands will at length be obtained, greatly superior in point of accuracy to any that is now extant.

One additional instrument would certainly be wanted, that is, a zenith sector for the determination of the latitudes, when the operation came to be extended to any considerable distance from the parallel of Greenwich. But this would not be necessary at first; while such a one is preparing by Mr. RAMSDEN, and which he will no doubt render the compleatest thing of the kind, the operation should be continued in the parallel of Greenwich, or in the perpendicular of its meridian, quite to the western side of the island in the manner following.

In more than one place of this Paper we have had occasion to express our regret, that the recent series of triangles did not afford distances sufficiently great between points reciprocally visible, for the best application of the pole-star observations, to the determination of the differences of longitude. It is believed, that the observations themselves are extremely near the truth, but not wholly free from error; therefore, whatever this may amount to, on double or triple the distance it would certainly be reduced to one-half or one-third part.

Shooter's Hill, and Nettlebed Heights on the eastern skirt of Oxfordshire, are reciprocally visible at the distance of about forty-six or forty-seven miles from each other. Nettlebed Heights, and a thin clump of trees on the Gloucestershire range of hills, called *Paul's Epistle*, about two miles westward from *Frog Mill*, on the left-hand side of the road leading from thence to Gloucester, may likewise be seen from each other at

the

the distance of fifty or fifty-two miles. This last commands a most extensive prospect over the plain of the Severn and the Welch mountains to a great distance beyond it. *Pen-y-Voel Hill*, called also the Sugar-loaf of Abergavenny, in Monmouthshire, would become the third station to the westward; and two, or at most three, stations more would reach St. David's Head, opposite to Wexford in Ireland.

But let us suppose, in the first place, the series of triangles to be extended only to the third station, in all which space it would be wholly unnecessary to observe any latitudes; by the pole-star observations, repeated a sufficient number of times on both sides of the pole, at each of the stations, the length of the degree of a great circle, perpendicular to the meridian, and consequently the differences of longitude, would thereby be obtained to the utmost precision. A determination of this sort would absolutely be conclusive, with regard to the length of the vertical and radius of the parallel in the latitudes of the respective stations, ascertainable by their distances from the perpendicular to the meridian of Greenwich.

The second part of the operation would be that of carrying a series of triangles southward from *Pen-y-Voel Hill*, in the direction of its meridian to the British Channel; and afterwards extending these triangles in the usual manner over the whole south part of the island between Kent and the Land's-End.

If, besides the zenith sector, another circular instrument was provided, and some additional annual expence allowed, in order to accomplish more speedily so great and useful a work, at the same time that the operations to the southward were carrying on, the series of triangles, in the direction of the meridian of *Pen-y-Voel Hill*, should be continued to the northward throughout

out the extent of the island till it fell into the Murray Frith. A new meridian might then be taken more to the westward, perhaps that of Inverness, or some hill near it, whereby the series would be extended to the North Sea, bounding the coasts of Sutherland and Caithness.

It is unnecessary here to enter into any minute detail of what should be the succeeding parts to be carried preferably into execution, as things of this sort would naturally present themselves, in the course of such important operations, to those entrusted with the direction. It is however sufficiently obvious, that having, as above supposed, obtained the measure of a portion of the meridian amounting nearly to sixteen degrees of latitude in continuity, between the Pyrenean mountains and the northern extremity of Britain, or more than one-sixth part of the distance between the equator and the pole; the things of the next consequence to be obtained would be, the measures of the *radii* of the vertical and parallel in the lowlands of Scotland, that is, in the latitude of Edinburgh, and again at the northern coast. In each of these situations it is evident, that about three degrees of longitude might be measured with great exactness. At the north, for instance, *Cape Wrath* being made the central station, from thence the *Orkney Islands* to the eastward and *Butt of the Island of Lewes* to the westward, being distinctly seen, would consequently become the stations to the right and left.

With regard to the use of white lights, so indispensably necessary in all operations of this sort, no opportunities have yet offered of ascertaining with precision the immense distance to which they may be seen in favourable circumstances of the weather, and with sufficient elevation of the stations above the sea. Those commonly used in the recent operation were only

three or four inches in diameter, and the largest but six or seven. Augmented to nine or ten inches, exhibited on the top of one high hill, and observed from the top of another, when there is no moonlight, and no rain or fog, they would probably be seen eighty or a hundred miles. In short, wherever the most faint looming of the land in a very clear day can be discerned, the lights, from their extraordinary brilliancy, would undoubtedly be seen in a dark night, when the air was perfectly clear.

Hence, it will readily be conceived, how easily and accurately any trigonometrical operations that might be carrying on in England and Ireland at the same time might be connected with each other, by means of these lights, alternately exhibited and observed, for instance, on *Brach-y-pwl Point, Holyhead Hill*, and the *Isle of Man*, on one side; and again on the *mountains of Wicklow, hill of Howth, and mountains of Mourne*, on the other.

In the Paper of 1787, and again in this, we have had occasion to remark on the improbability of being able to determine the differences of longitude, by the instantaneous explosion of light, so accurately as by angular measurement with a fine instrument, applied as it has been in the recent operation. But since, undoubtedly, there will be different opinions on this head, it will be very proper that both modes should be tried, that the results may be compared.

To the eastward of Greenwich, the station for explosion might be taken at *Montlambert, Fienne Windmill*, or at *Folkstone Turnpike*, in order to render the distance of the extreme stations as great as possible. Any of these points would be visible from *Crowborough Beacon*, which would become the station of the English astronomer with his clock and instruments. That of the French astronomer would of course be taken

taken in the most convenient place inland on the range of chalk hills, visible from the place of explosion, and the easiest connected with the triangles of the meridian of Paris in the neighbourhood of *Helfaut* and *Bouvigny*. *Crowborough* is about 70 miles distant from *Montlambert*, and a point in the direction of these two near *Helfaut* would be about 32 miles inland from *Montlambert*, which would give for the extreme distance about 102 miles.

If experiments of the same kind were to be made to the westward of Greenwich, those very stations, already proposed for the continuation of the triangular operation, would be the fittest that could be chosen for the purpose.

Now, supposing the operations already mentioned in the parallel of Greenwich to be executed, the meridian of Pen-y-Voel Hill extended to the northern extremity of Scotland, and three degrees of longitude measured in that latitude, while the East India Company were carrying on operations of the same nature on the coast of Coromandel and in Bengal, every thing would then be done that Britain could do within her own dominions, in regard to the determination of the figure and dimensions of the earth. If, after this, any doubts remained, these might easily be removed, by Portugal's measuring a degree or two of the meridian under the equator, and also a portion of the earth's equatorial circumference; while some other nation repeated the operations at the polar circle, or made new ones still nearer to the pole, if such should be found practicable, all which has been suggested in the Paper of 1787. For the farther illustration of this subject, it will be proper to refer to fig. 3. in Plate X.

With regard to the execution of any future operation that may, and which it is hoped will, be hereafter undertaken in

Britain, there remains but one point more to be mentioned, that is, the measurement of new bases. In the execution of the great map of France, no fewer than seventeen were measured. But, with such instruments as have been used in this country, a smaller number would suffice; and the best situations for the purpose will naturally present themselves in the course of the operations.

Those that immediately occur to the Writer of this Memoir, as likely to be found the most proper, are the following, *viz.*

1. On Sedgemoor in Somersetshire.
2. On Boston Fens in Lincolnshire.
3. On the sands on the coast of North Wales
between Pen man Mawr and Beaumaris, . . .
4. On the sands between Holy Island and
Berwick upon Tweed, . . .
5. On Kincairden and Flanders Moss, westward from Stirling.
6. On the sands on the coast of Aberdeenshire
between the mouth of the Don and Newburgh, . . .
7. The sands on the coast of Murray, between
the mouths of the rivers Findhorn and Nairn, . . .
8. On the Moan morasses, inland from the Whiten Head, on the coast of Sutherland.

In the measurement of these bases, which should not be less than six, but as often as possible even eight or ten miles in length, there would not be any necessity for that wonderful exactness that was requisite for determining the length of the first and second bases on Hounslow Heath and Romney Marsh. Supposing them to be executed with the steel chain alone within a few feet of the truth, it would be perfectly sufficient to shew

that no error of any consequence had accumulated in carrying on the operation to these distant points respectively, even as far as the remote shore of the Northern Ocean.

Finally, in order to preserve the primitive scale of distances, whereon the accuracy of the recent operation, and all future ones that may hereafter be connected with it, must always be supposed to depend, it is indispensably necessary to establish, without loss of time, some permanent marks at the extremities of the base on Hounslow Heath*. These should be low circular buildings, rising but a few feet above the surface of the Heath, composed of the hardest materials, such as granite, and constructed in the most durable manner by dove-tailing the stones into each other. They would resemble those basements of ancient crosses we often meet with, formed into regular steps, whereby the ascent is rendered easy to the top of a circular table or platform, of sufficient dimensions for the reception of the great instrument on any future occasion.

In the interior part of these little buildings, metal tablets would be inserted, containing the name of that much-beloved Monarch in whose reign the operation was begun, and these buildings executed; the distance from one to the other; the angle of the base with the meridian; and also the magnetical variation.

It is not to be doubted, that the respective lords of the manors will readily vest in the Royal Society, the property of the two small spots of the Heath sufficient for the erection of these

* Soon after the measurement of the base, Mr. MYLNE, F. R. S. at my desire, was so obliging as to give a design for a building of this kind, which, being constructed nearly in the way of the Eddystone Light-house, executed by the ingenious Mr. SMEATON, would answer very well.

termini. They should be carried into immediate execution; for, if this business should be postponed for any length of time, there will be danger of its being altogether forgotten. In a few years the wooden pipes sunk in the earth will decay; and thus the primitive scale of distances, which cost so much labour and expence to obtain, will be irretrievably lost.

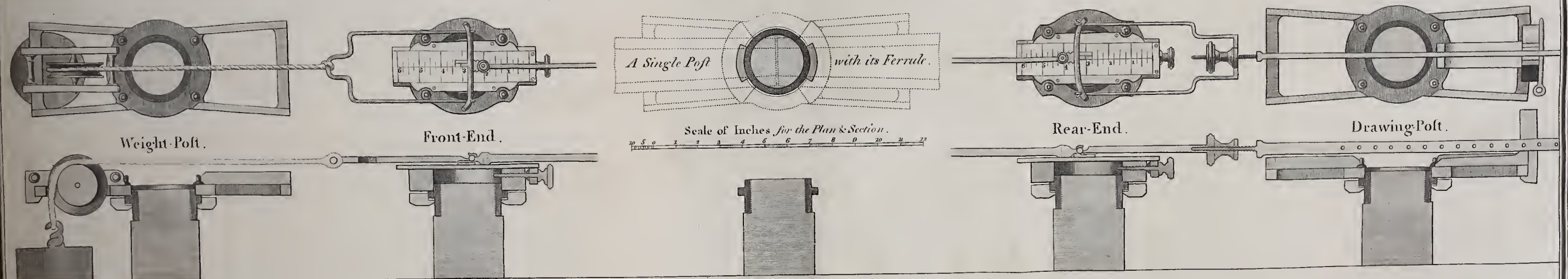


Arrangement of the Posts for each Space of 100 Yards, or Length of 3 Chains.

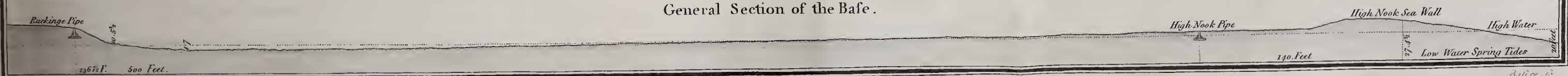
Elevation of the Coffering for each Chain.

Plan of the Coffering for each Chain.

Plan & Section of the Apparatus for the Extremities of the Chain; Scale $\frac{1}{4}^{\text{th}}$ part of the real dimensions.

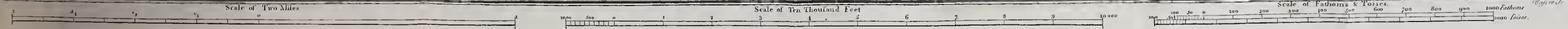


General Section of the Base.

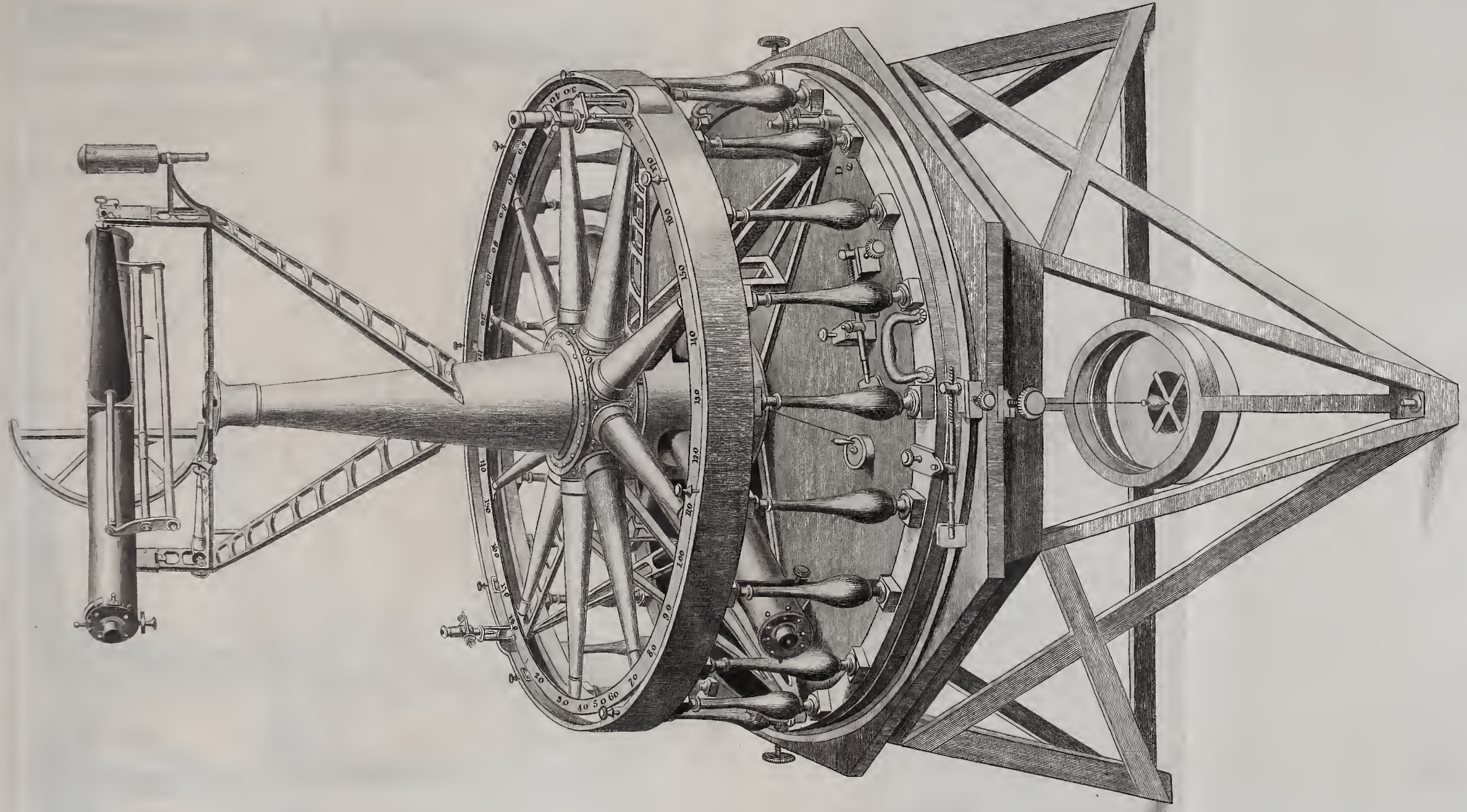


PLAN shewing the situation of the BASE of VERIFICATION measured in ROMNEY MARSH in the Autumn of 1787.

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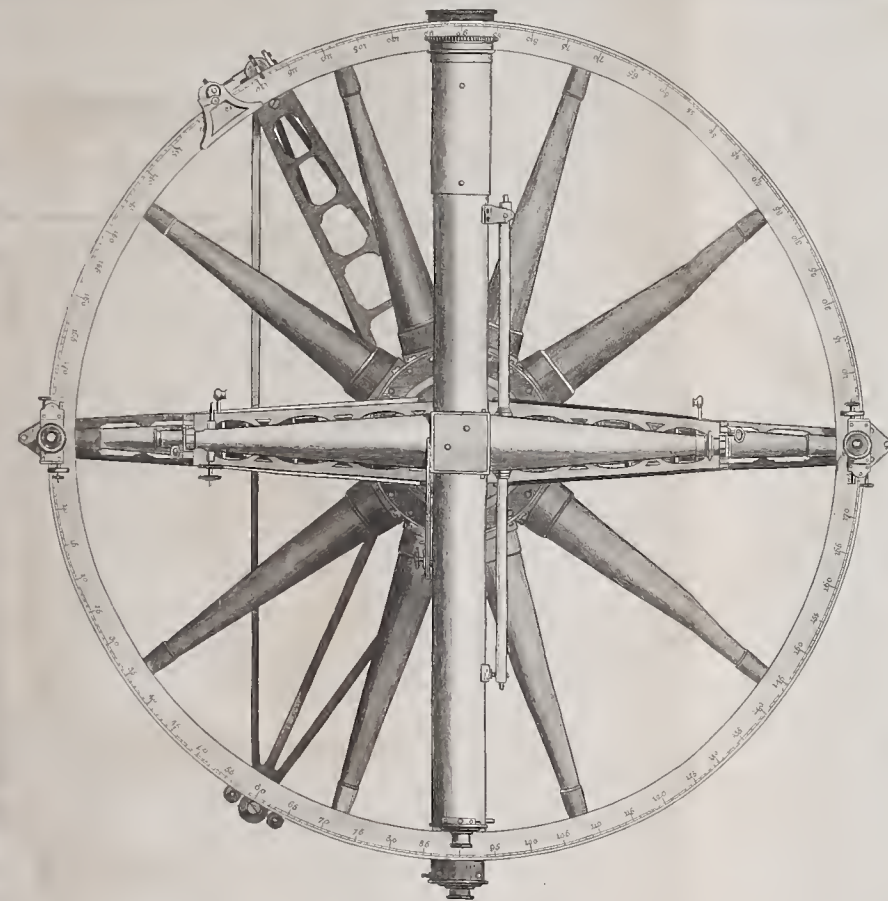


General View of the Instrument.

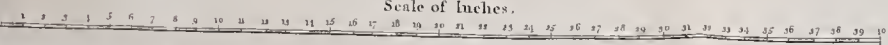
J. Moline del.

Barrow sc.

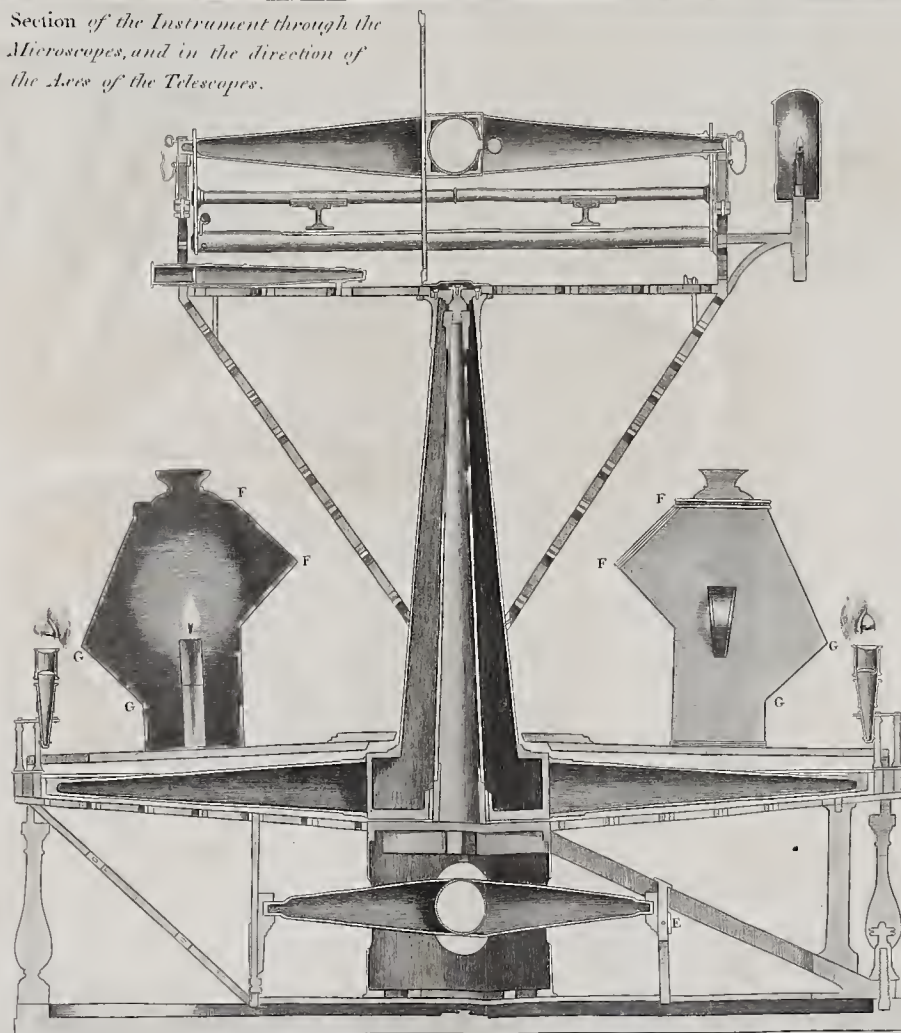
Plan of the Instrument.



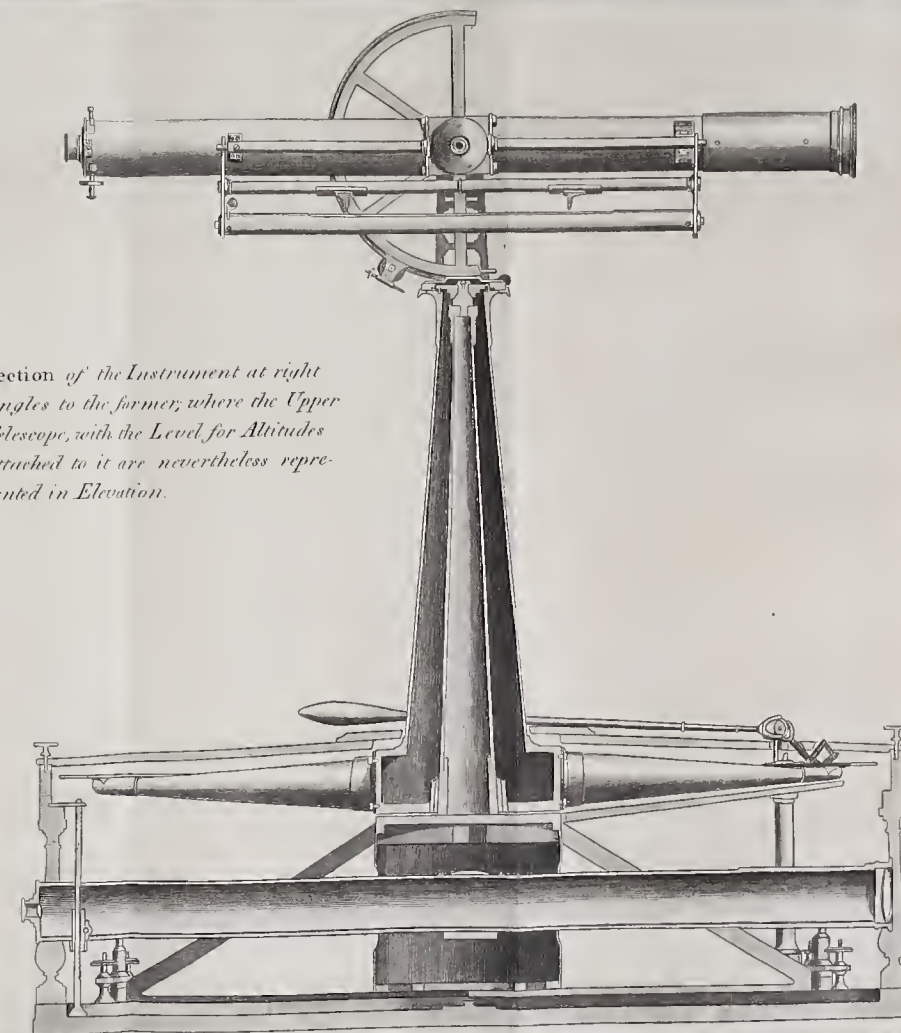
Scale of Inches.



Section of the Instrument through the Microscopes, and in the direction of the Axes of the Telescopes.

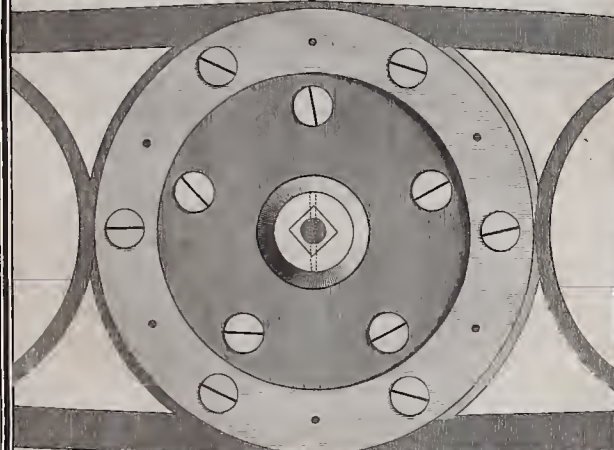


Section of the Instrument at right angles to the former, where the Upper Telescope, with the Level for Altitudes attached to it are nevertheless represented in Elevation.

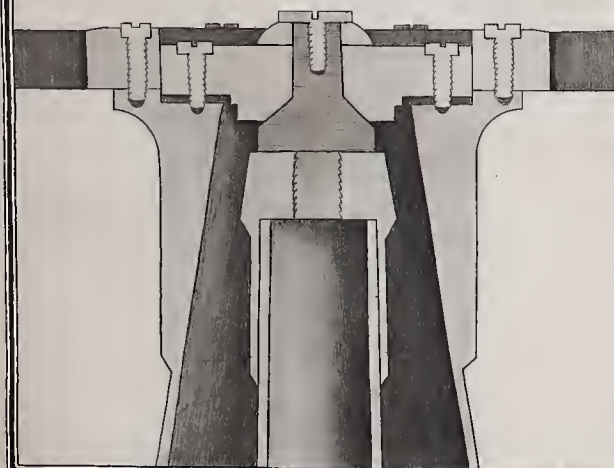




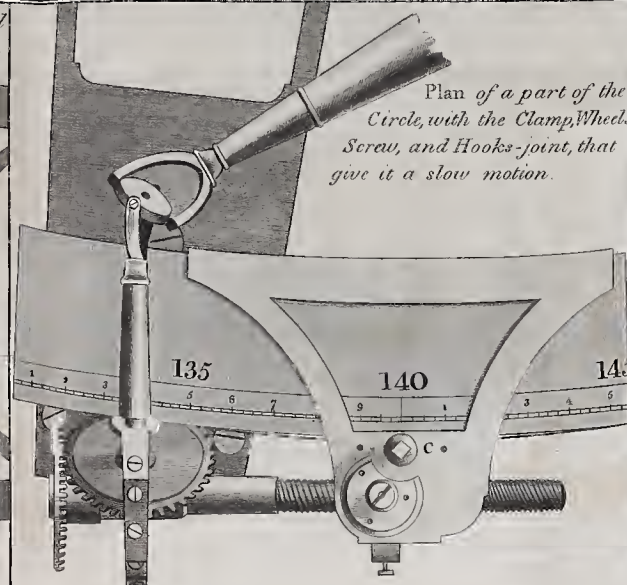
Plan of the Top of the vertical Axis, with part of the horizontal Bar for carrying the Upper Telescope, in their real dimensions.



Section of the Top of the vertical Axis.



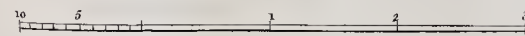
Plan of a part of the Circle, with the Clamp, Wheels, Screw, and Hooks-joint, that give it a slow motion.



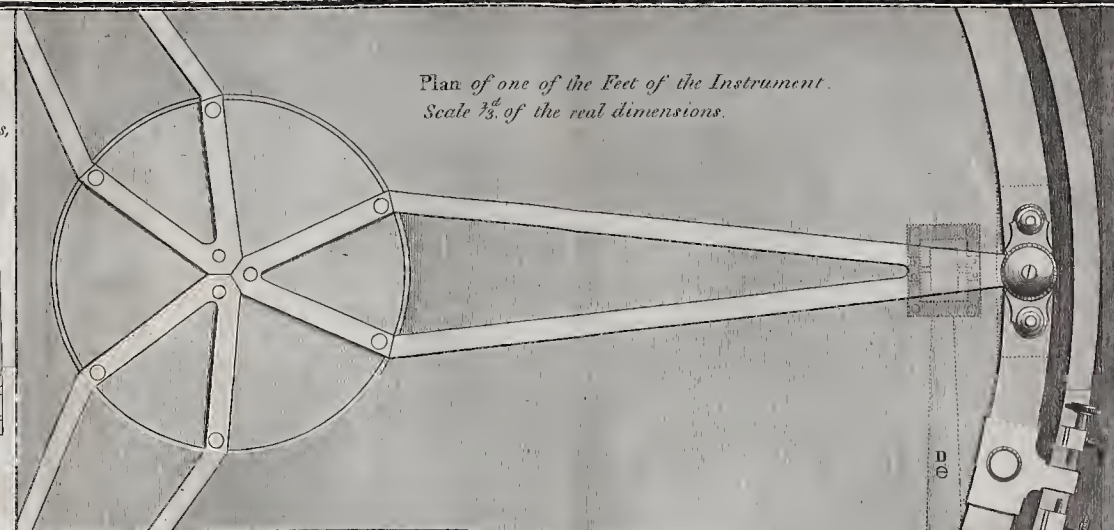
Milled-head-Key placed at C for clamping the Instrument.



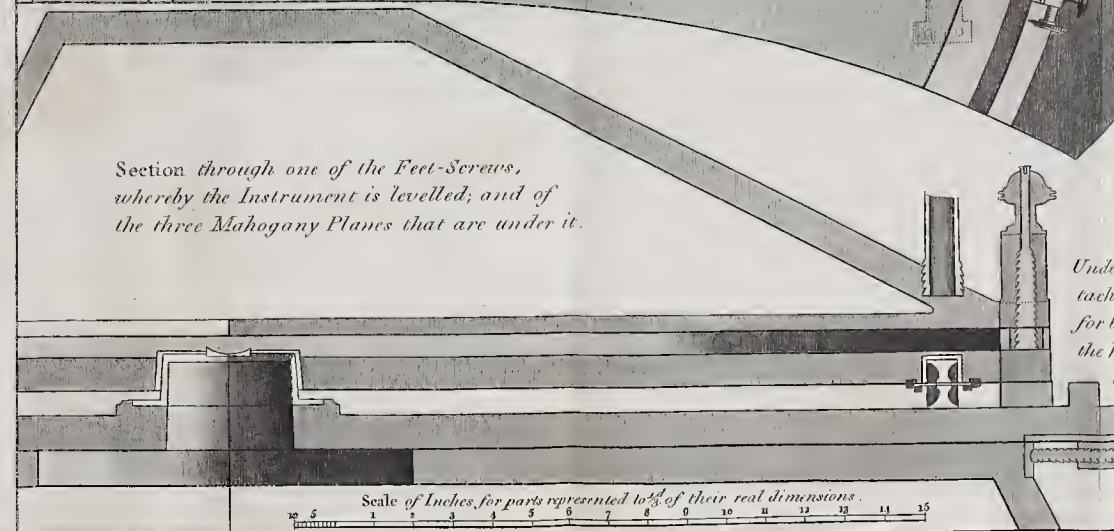
Scale of Inches for parts represented in their real dimensions.



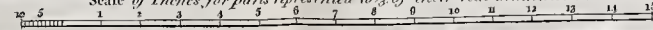
Plan of one of the Feet of the Instrument. Scale $\frac{1}{3}$ of the real dimensions.



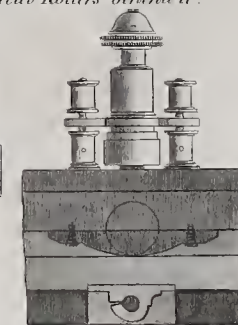
Section through one of the Feet-Screws, whereby the Instrument is levelled, and of the three Mahogany Planes that are under it.



Scale of Inches for parts represented to $\frac{1}{2}$ of their real dimensions.



Front Elevation of one of the Feet-Screws, shewing likewise one of the Blocks of box-wood, and one of the conical Rollers behind it.



Front Section of the Spring, and Conical Roller.

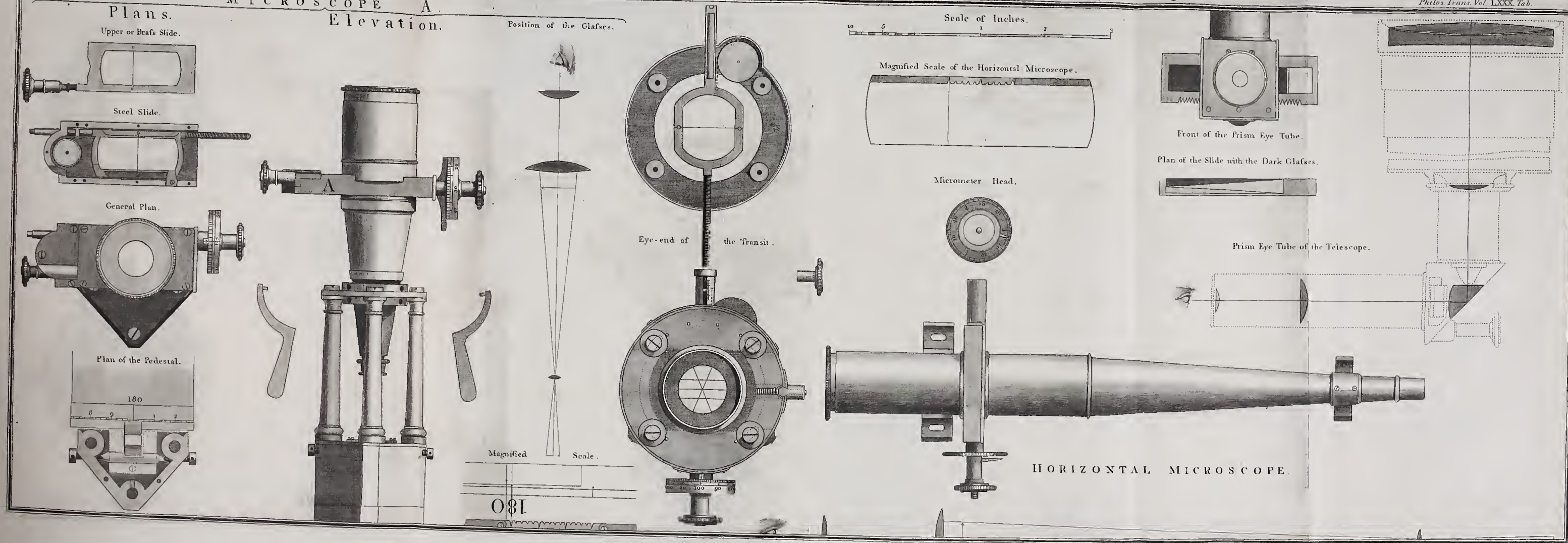


D

Under side of one of the Four Screws attached to the octagonal mahogany Plane for bringing the Center of the Instrument by the help of its plummet, over any point on the ground.



For the Explanation of the Microscopes, and Mechanism for the Wires in the Eye-ends of the Telescopes.



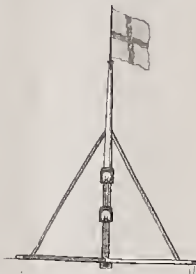
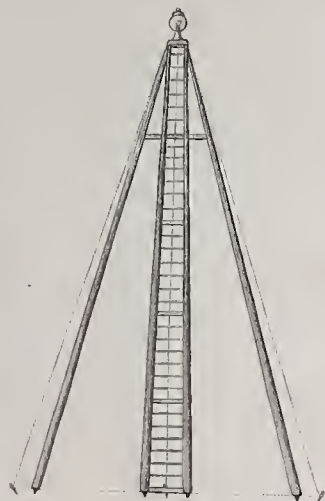
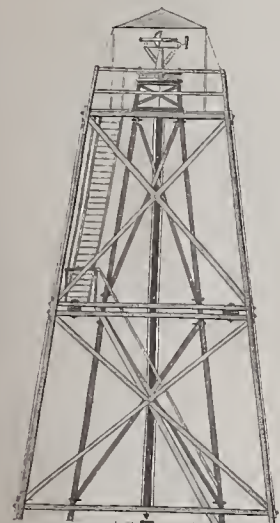
Portable Scaffold

Tripod Ladder

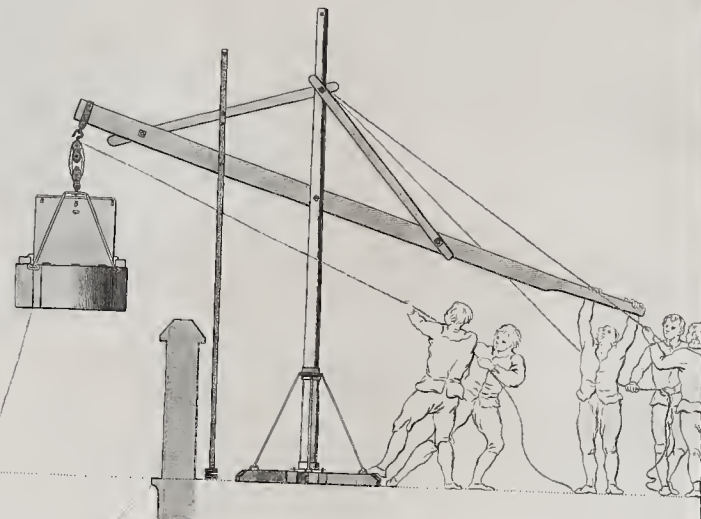
Flagstaff
carrying likewise
Reverberatory Lamps.

Portable Crane

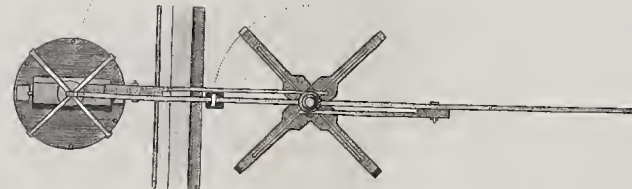
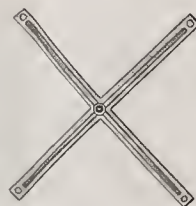
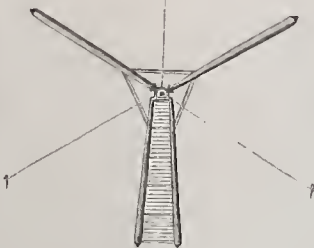
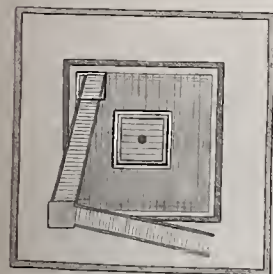
Elevations



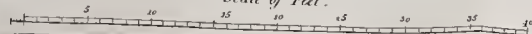
Tripod
for White Lights.



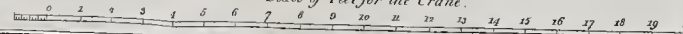
Plans

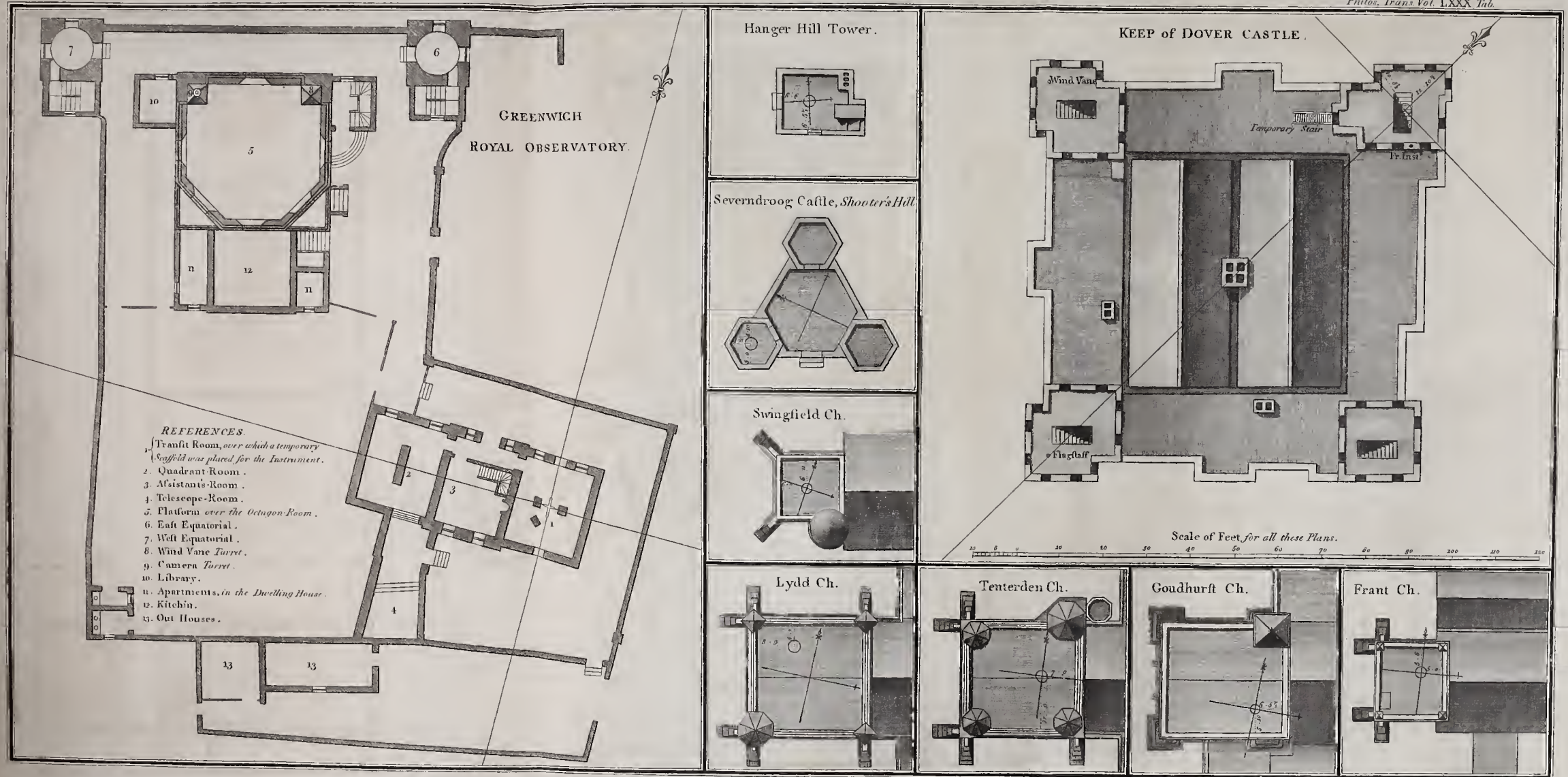


Scale of Feet.



Scale of Feet for the Crane.

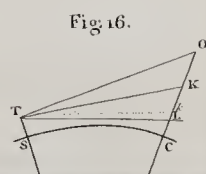
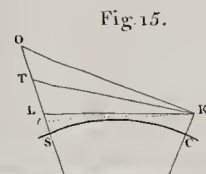
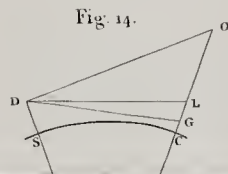
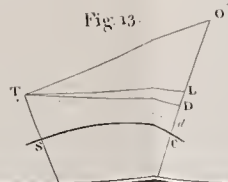
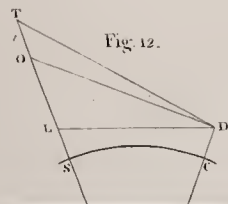
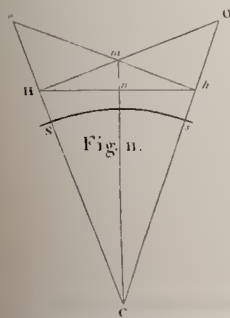
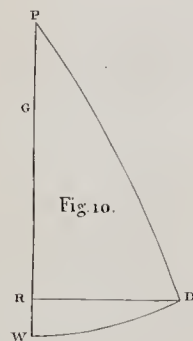
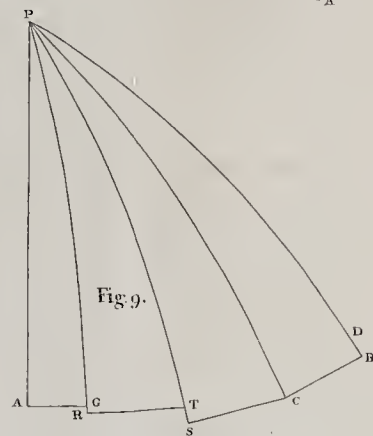
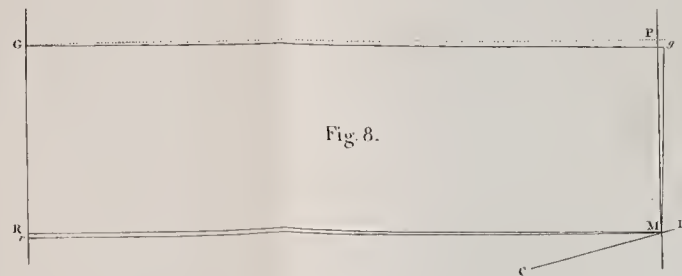
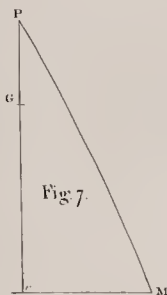
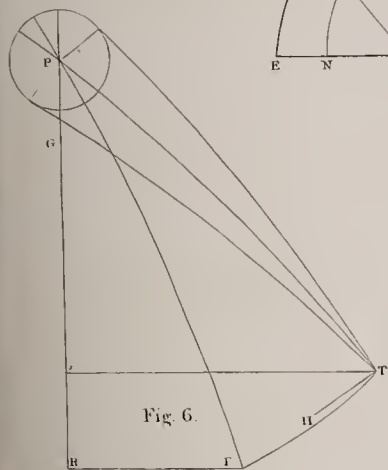
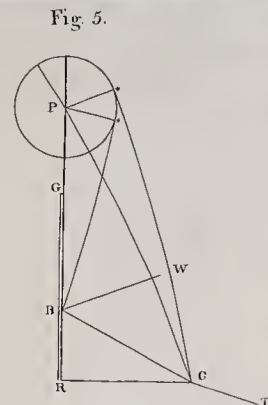
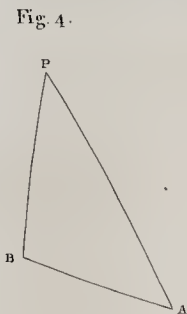
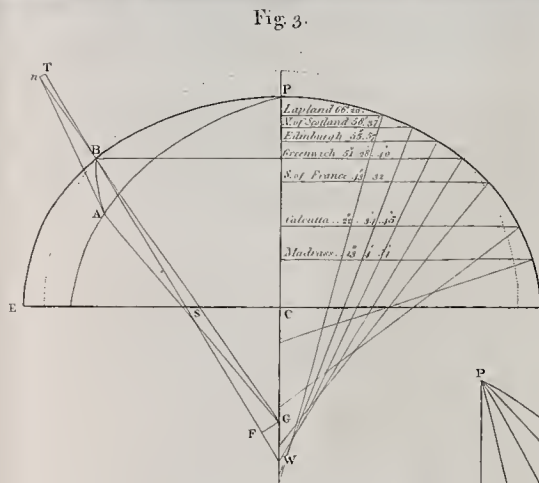
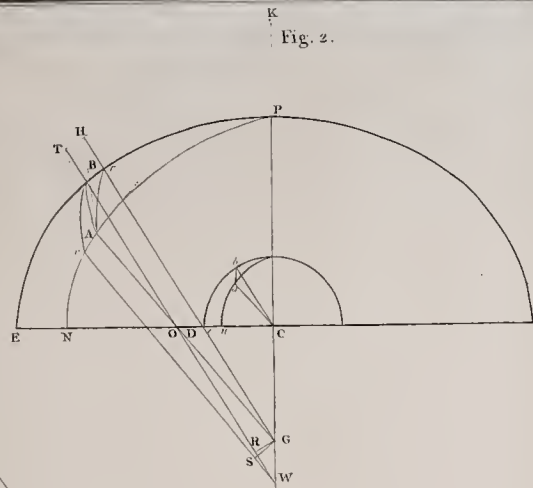
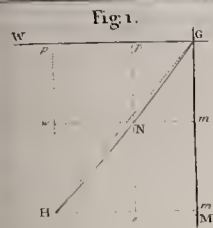


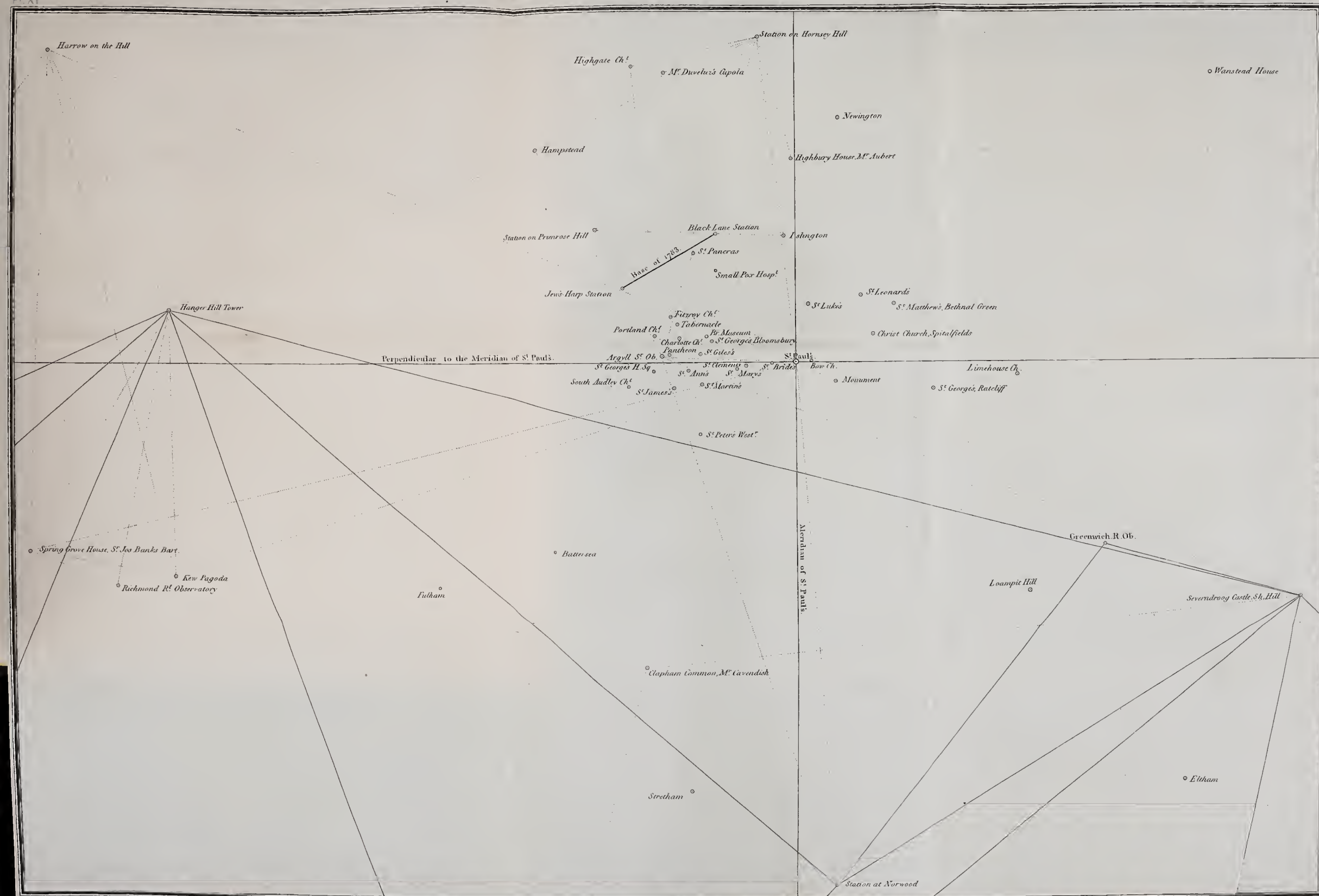


PLAN of the TRIANGLES whereby the DISTANCE between the ROYAL OBSERVATORIES of GREENWICH and PARIS has been determined

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Scale of Thirty Thousand Feet

Scale of Six English Miles

